Do the "Three-Point Victory" and "Golden Goal" Rules Make Soccer More Exciting?

Isabelle Brocas and Juan D. Carrillo

Journal of Sports Economics 2004; 5; 169

DOI: 10.1177/1527002503257207

Do the "Three-Point Victory" and "Golden Goal" Rules Make Soccer More Exciting?

ISABELLE BROCAS
JUAN D. CARRILLO
University of Southern California and CEPR

This article argues that a rigorous application of simple game theory tools may provide unambiguous predictions about the behavior of teams in sports. As an illustration, the authors analyze the merits of two controversial changes in soccer rules, namely, the "three-point victory" and the "golden goal." Building on well-accepted premises, the authors show that contrary to the common belief, the incentives of teams to play offensively may be lower under the three-point victory than under the traditional two-point victory. They also provide clear and simple recommendations for the improvement of these rules.

Keywords: game theory; design of sport rules; soccer

MOTIVATION

A major criticism of applied game theory is that it often generates results that are either obvious or inconclusive. Most games played in real life are complex, with multidimensional strategies and incomplete information. Besides, the pay-offs in some states are often ex-ante unspecified. The assumptions needed to have a well-defined game and to avoid a multiplicity of equilibria tend to make the models excessively simplistic and/or the theoretical conclusions trivial.

One area in which simple game theoretic tools can be used satisfactorily to predict behavior is sport. There are concrete situations in sport in which the game is simple and well defined and in which players choose strategies in a known and small set. Few authors have recognized this advantage of sport. Notable exceptions are the recent articles by Walker and Wooders (2001), Chiappori, Levitt, & Groseclose (2002), Palacios-Huerta (in press) and Palomino,

AUTHORS' NOTE: *We thank Micael Castanheira and an anonymous referee for their comments.*

JOURNAL OF SPORTS ECONOMICS, Vol. 5 No. 2, May 2004 169–185
DOI: 10.1177/1527002503257207

Rigotti, and Rustichini (1999). The first article looks at serve-and-return play at Wimbledon. The second and third analyze penalty kicks in European soccer leagues. All three show that the data support the mixed strategy Nash equilibrium prediction of game theory. The fourth article also deals with soccer. It shows that although the behavior of teams is roughly consistent with rationality (losing teams adopt more offensive strategies than do winning teams), there is still a substantial component of irrationality or "passion," illustrated by the fact that teams perform better at home than they do away.

Although highly unintuitive and a major breakthrough for the development of the theory of games, the theory behind the minimax theorem has become standard in economics. The fact that the observed behavior is consistent with the fundamental insights of noncooperative game theory (as these articles demonstrate) is reassuring: It validates to a certain extent the use of theoretical models to describe and predict individual decision making. The strength of these articles relies on their ability to identify games where predicting the optimal strategy is very difficult for people unfamiliar with game theory but trivial for anyone with a minimum training in formal economics.

Starting from the conclusion that game theory usefully describes strategic interactions between individuals, this article has a different objective. First, we want to design a simple theoretical model that builds on standard and widely accepted premises and yet delivers theoretical predictions that are clear and simple to understand but at the same time are original and different from the common wisdom. Second, using the insights of our model, we propose simple changes in the rules that result in unambiguous improvements.

To focus the discussion, we concentrate on soccer and study the effects of the two major changes in rules introduced in international events such as the World Cup and the European Cup in the 1990s: the "three-point victory" (3PV) and the "golden goal" (GG). The 3PV system is used in league tournaments. Under this rule, the winner of a match obtains three points and the loser obtains zero points. In the case of a draw, each team obtains one point. The main argument in favor of this system—instead of the traditional two-point victory (2PV)—is simple. Adopting an offensive strategy increases the team's chances of scoring but also of conceding a goal. Therefore, teams are encouraged to play more offensively if the expected pay-off of breaking a tie is raised. The GG rule is used in elimination tournaments. Before its adoption, if two teams were tied at the end of the regular time, they would play a fixed 30 minutes overtime, and if the draw persisted, they would proceed to the penalty kicks. With the GG rule, the first team to score within the 30 minutes of overtime wins the match. If no one scores, the penalty kick method again determines the winner. Thus, the GG rule decreases the expected time of play and, other things being equal, the probability of reaching the penalty kick stage.

In this article and with the help of basic game theory, we qualify the ideas stated previously in favor of 3PV and GG. We show that although correct, the arguments are excessively simplistic because they only capture one effect of the rule on the behavior of teams. More specifically, we show in Proposition 1 that conditional on the game being tied, increasing the value of a victory will induce teams to adopt a more offensive strategy toward the end of the game in order to break the tie in one direction or another late in the match. However, under some conditions, it will also induce teams to use a more defensive strategy toward the beginning of the game in order to avoid being led early in the match and therefore keep the option of trying to break the tie late in the match. As a result, teams may on average play more defensively under 3PV than under 2PV. In other words, just by accounting for the possibility of changing the strategy over time, we show that a rule established to favor certain objectives (in this case, a more offensive play) may in fact be counterproductive using that same criterion. In Proposition 2, we argue that in the context of elimination tournaments, the GG rule modifies the pay-off of scoring (it prevents the team that concedes a goal to come back on the game) but not the incentives of teams to play offensively. Therefore, the popular idea that the introduction of the GG rule did not affect the strategy of teams and reduced the likelihood of reaching the penalty kicks stage is supported by our model. However, to extract the best from this rule, we need to merge it with the 3PV: As shown in Proposition 3, the combination of an increase in the expected value of breaking a tie (3PV rule) together with a reduction in the ability to come back in the game when the opponent scores (GG rule) is unambiguously beneficial for the game: It always induces teams to play more offensively than under 3PV alone. It is interesting that this possibility has never been considered in practice despite the simplicity of its implementation. To sum up, this article shows that basic game theory principles can be a powerful tool to obtain nontrivial theoretical insights about the behavior of players in sports. Moreover, a careful modeling can deliver unambiguous recommendations for the improvement of existing rules.

Before presenting our formal model, we would like to mention two other articles indirectly related to ours. Lazear and Rosen (1981) were among the first authors who used a game theoretic model to analyze the optimal design of tournaments. The article focuses on the incentives of players to exert costly effort as a function of the type of tournament (rank-order versus linear score differences). It shows that both reward schemes can induce the same (first-best) level of effort if teams are homogenous and risk neutral. Chan, Courty, and Hao (2001) consider a dynamic version of that model and show the superiority of linear schemes. However, if the public has a preference for uncertainty in the final outcome (which they label a "demand for suspense"), then rank-order tournaments may become preferable. The major difference between these articles and ours is that they focus on the incentives to exert costly effort, whereas we concentrate on the incentives for the strategic allocation of effort between offense and defense.

A MODEL OF THE 3PV RULE

Strategies of Teams and Timing

We consider the simplest model needed to capture the main effects of the scoring system on the strategy of teams. Two teams, $i \in \{A, B\}$, play a match against each other. The winner of the game gets x (≥ 2) points, and the loser gets zero points. In the case of a draw, they both get one point. Teams are risk neutral and play in a league tournament. Their objective is to maximize the expected number of points collected in the game.[1]

For simplicity, we assume that each team i decides at the beginning of the game (date $t = 1$) and at half-time (date $t = 2$) the strategy θ_k^i to be employed during the upcoming half period. Final pay-offs are realized at the end of the match (date $t = 3$). The parameter θ_k^i denotes the degree of "offensive" play by team i at date t, with higher values of θ denoting a more offensive strategy.[2] This value is selected by each team from the same compact set $\Theta = [\underline{\theta}, \overline{\theta}]$. Playing more offensively increases the chances of scoring (and therefore the probability of winning the match) but also the chances of conceding a goal (and therefore the probability of losing the match). Naturally, the optimal strategy of each team will be contingent on the score at the time of selecting it.

Denote by $\tau \in \{1, 2\}$ the two half periods of play, that is, $\tau = 1$ refers to the first half period (between $t = 1$ and $t = 2$) and $\tau = 2$ refers to the second half period (between $t = 2$ and $t = 3$). Suppose that during each half period τ, only three events, $e_\tau \in \{a, o, b\}$ concerning the score of the game may occur: team A scores either one more goal ($e_\tau = a$), the same number of goals ($e_\tau = o$), or one less goal ($e_\tau = b$) than team B. The relative likelihood of these events will depend on the strategies (θ^A, θ^B) selected by both teams. From now on, we will call dates ($t \in \{1, 2, 3\}$) the beginning, half time, and end of the match and half periods ($\tau \in \{1, 2\}$) the intervals of play going from beginning to half time and from half time to the end of the match. The timing of the game can thus be summarized in Figure 1.

We analyze this game using a reduced-form model. Instead of defining the probability that each team scores a goal given both teams' strategies, we work directly with the probability that each team scores one more goal than its rival in each half period (events $e_\tau = a$ and $e_\tau = b$) given the strategies θ^A and θ^B of both teams. Denote

$$\alpha\,(\theta^A, \theta^B) = \Pr(a \mid \theta^A, \theta^B) \text{ and } \beta\,(\theta^A, \theta^B) = \Pr(b \mid \theta^A, \theta^B).$$

We use the subscript n in $\alpha(\bullet)$ and $\beta(\bullet)$ to denote the partial derivative with respect to the n^{th} argument. These probabilities satisfy the following assumptions.

Assumption 1: (a) $\alpha_1(\theta', \theta'') > 0$; $\alpha_{11}(\theta', \theta'') \leq 0$; $\alpha_2(\theta', \theta'') > 0$; $\alpha_{22}(\theta', \theta'') > 0 \ \forall\, \theta', \theta''$
(b) $\beta_1(\theta', \theta'') > 0$; $\beta_{11}(\theta', \theta'') > 0$; $\beta_2(\theta', \theta'') > 0$; $\beta_{22}(\theta', \theta'') \leq 0 \ \forall\, \theta', \theta''$

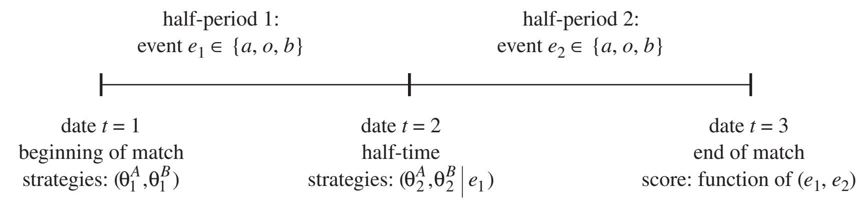

Figure 1: Timing of the Game

Assumption 1 is quite uncontroversial. It simply states that choosing a more offensive strategy (higher θ) increases both the chances of scoring and the chances of conceding one more goal than the rival.[3] Furthermore, the marginal probability of scoring one more goal than the rival is decreasing in the level of offensive play, and the marginal probability of conceding one more goal than the rival is increasing in the level of offensive play. The conditions on the second derivatives will ensure the concavity of the overall maximization problem.

Assumption 2: $= \alpha_{12}(\theta', \theta'') = \beta_{12}(\theta', \theta'') = 0 \ \forall \ \theta', \theta''$
Assumption 3: $\alpha(\theta', \theta'') = \beta(\theta'', \theta') \ \forall \ \theta', \theta''$

Assumption 2 states that the marginal effect of one team's level of offensive play is independent of the strategy employed by the other team. Assumption 3 states that teams are homogeneous, that is, equally strong.[4] Assumptions 2 and 3 are debatable to say the least. Homogeneity is more the exception than the rule: Few matches are played by teams of equal strength. As for the marginal effect of one team's strategy, it will often depend on the behavior of its opponent, although there is no consensus on its sign.[5] However, there are two interconnected reasons for maintaining these assumptions, the first of which is simplicity. Under these assumptions, we will obtain an equilibrium that is unique and easy to characterize. This will allow us to perform clear-cut comparative statistics about the effect of the reward system on the strategy of teams. Second and more important is transparency. Strategies can always have perverse indirect effects in pay-offs if we include some suitably chosen asymmetries in the teams and/or if we "twist sufficiently" the second and cross-derivatives of the scoring probabilities. This is not the purpose of our article. Instead, we present a model that captures in the most uncontroversial way the strategic choice of teams in a soccer game. Then, we introduce as our only departure the ability of teams to change their strategy during the game (or more accurately, at half time).

VALUE FUNCTIONS OF TEAMS AND ASSOCIATED PAY-OFFS

The pay-off of each team depends exclusively on the final score at date $t = 3$, that is, on the realization of the stochastic events e_1 and e_2. If e_1 and e_2 are such that a occurs more often than b (both equally often, respectively; b more often than a, respectively), then team A wins (ties and loses, respectively) the match, in which case its pay-off is x (1 and 0, respectively) and the pay-off of team B is 0 (1 and x, respectively).

Denote by $v_1^i(\theta_1^A, \theta_1^B)$ the value function of team i at the beginning of the match ($t = 1$) if teams select strategies θ_1^A and θ_1^B for the first half period. Similarly, $v_2^i(\theta_2^A, \theta_2^B | e_1)$ denotes the value function of team i at half-time ($t = 2$) given the current score (i.e., the realization of event e_1 during the first half period) if strategies θ_2^A and θ_2^B are selected for the second half period. This game is solved by backward induction and using the subgame perfect equilibrium concept. The value function of team A at half time is as follows (the value function of B is determined in a similar way):

$$v_2^A\left(\theta_2^A, \theta_2^B \mid a\right) = \left[1 - \beta\left(\theta_2^A, \theta_2^B\right)\right]x + \beta\left(\theta_2^A, \theta_2^B\right)$$

$$v_2^A\left(\theta_2^A, \theta_2^B \mid b\right) = \alpha\left(\theta_2^A, \theta_2^B\right)$$

$$v_2^A\left(\theta_2^A, \theta_2^B \mid o\right) = \alpha\left(\theta_2^A, \theta_2^B\right)x + \left[1 - \alpha\left(\theta_2^A, \theta_2^B\right) - \beta\left(\theta_2^A, \theta_2^B\right)\right]$$

In words, if A is leading at half time ($e_1 = a$), then its value function $v_2^A(\cdot \mid a)$ is the probability of not receiving one more goal than its opponent $(1 - \beta)$ times the pay-off in case of victory (x) plus the probability of receiving one more goal than its opponent (β) times the payoff in case of a draw (1). The same logic applies to $v_2^A(\cdot \mid b)$ and $v_2^A(\cdot \mid o)$. Using these value functions, we can then determine the equilibrium strategies of teams for the second half period (θ_2^A, θ_2^B) conditional on the event realized during the first half period ($e_1 \in \{a, o, b\}$):

$$\underline{\theta} = \underset{\theta}{\arg\max}\; v_2^A\left(\theta, \theta_2^B \mid a\right) = \underset{\theta}{\arg\max}\; v_2^B\left(\theta_2^A, \theta \mid b\right)$$

$$\bar{\theta} = \underset{\theta}{\arg\max}\; v_2^A\left(\theta, \theta_2^B \mid b\right) = \underset{\theta}{\arg\max}\; v_2^B\left(\theta_2^A, \theta \mid a\right)$$

$$\theta^{**} = \underset{\theta}{\arg\max}\; v_2^A\left(\theta, \theta_2^B \mid o\right) = \underset{\theta}{\arg\max}\; v_2^B\left(\theta_2^A, \theta \mid o\right)$$

where given Assumptions 1, 2, and 3, the strategy θ^{**} is unique and solves the following:[6]

$$\frac{\alpha_2\left(\theta^{**}\right)}{\alpha_1\left(\theta^{**}\right)} = x - 1 \tag{1}$$

When a team is leading at half time, it will choose the most defensive strategy ($\underline{\theta}$) during the second half period in order to minimize the probability of conceding a goal. Conversely, when a team is losing at half time, it will only be interested in maximizing its probability of scoring one more goal than the rival; therefore, it will choose the most offensive strategy ($\bar{\theta}$). The most interesting situation arises when the score is tied at half time. In this case, the optimal second period strategy of both teams is given by Equation 1. The interpretation is simple. Each team sets its optimal level of offensive play for the second half period at the value where the marginal increase in the probability of a victory [$\alpha_1(\bullet)$] weighted by the absolute increase in the pay-off [$x - 1$] equals the marginal increase in the probability of a defeat [$\beta_1(\bullet)$] weighted by the absolute decrease in the pay-off [1]. Given teams' homogeneity, $\beta_1(\theta') \equiv \alpha_2(\theta')$ and θ^{**} follows.[7] Differentiating Equation 1, we notice that

$$\frac{\partial\theta^{**}(x)}{\partial x} = \frac{\alpha_1\left(\theta^{**}\right)}{\alpha_{22}\left(\theta^{**}\right) - (x-1)\alpha_{11}\left(\theta^{**}\right)} > 0 \tag{2}$$

Other things being equal, if teams are tied at half time, they will play more offensively during the second half period the greater is the value of a victory x. In fact, Equation 2 formalizes the standard (static) argument in favor of the 3PV relative to the 2PV system: By increasing the expected pay-off of breaking a tie, teams are encouraged to take more risks, that is, to adopt more offensive strategies.

As discussed in the introduction, our goal is not to refute this argument. On the contrary, we take this theory as our starting point and build on it. Yet, we claim that a static analysis may not be appropriate for this game. In other words, we ask whether the conclusion presented in Equation 2 holds when we assume that the game is dynamic and that strategies can be modified over time. The simplest way to answer this question is to study the two half-period models previously depicted. Having analyzed the Nash equilibrium of the second half-period subgame contingent on the score at half time, we now proceed by backward induction and determine the equilibrium strategy for the first half period selected at the beginning of the game. Naturally, teams are tied when the match starts. The value function of team A at the beginning of the match is as follows:

$$\begin{aligned} \boldsymbol{v}_1^A\left(\theta_1^A,\theta_1^B\right) = {} & \alpha\left(\theta_1^A,\theta_1^B\right)\boldsymbol{v}_2^A\left(\underline{\theta},\bar{\theta}\,|\,\boldsymbol{a}\right) + \left[\mathbf{1} - \alpha\left(\theta_1^A,\theta_1^B\right) - \beta\left(\theta_1^A,\theta_1^B\right)\right]\boldsymbol{v}_2^A\left(\theta^{**},\theta^{**}|\boldsymbol{o}\right) \\ & + \beta\left(\theta_1^A,\theta_1^B\right)\boldsymbol{v}_2^A\left(\bar{\theta},\underline{\theta}\,|\,\boldsymbol{b}\right) \end{aligned}$$

The equilibrium strategy for the first half period then is as follows:

$$\theta^* = \underset{\theta}{\arg\max}\ v_1^A\left(\theta,\theta_1^B\right) = \underset{\theta}{\arg\max}\ v_1^B\left(\theta_1^A,\theta\right)$$

Given Assumptions 1, 2, and 3 and the same reasoning as in Equation 1, θ* is unique and solves the following:

$$\frac{\alpha_2\left(\theta^*\right)}{\alpha_1\left(\theta^*\right)} = \frac{v_2^A\left(\underline{\theta},\bar{\theta}\,|\,a\right) - v_2^A\left(\theta^{**},\theta^{**}\,|\,o\right)}{v_2^A\left(\theta^{**},\theta^{**}\,|\,o\right) - v_2^A\left(\bar{\theta},\underline{\theta}\,|\,b\right)} \equiv \frac{(x-1)g+h}{g+(x-1)h} \tag{3}$$

where $g = 1 - \beta(\underline{\theta}, \bar{\theta}) - \alpha(\theta^{**}, \theta^{**})$ and $h = \beta(\theta^{**}, \theta^{**})$. Differentiating Equation 3, we have the following:

$$\frac{\partial\theta^*(x)}{\partial x} = \frac{\alpha_1(\theta^*)(g+h)\left[g - h - \dfrac{\partial\theta^{**}(x)}{\partial x}x(x-2)(\alpha_1(\theta^{**})+\alpha_2(\theta^{**}))\right]}{\left[\alpha_{22}(\theta^*)\left(g+(x-1)h\right) - \alpha_{11}(\theta^*)\left((x-1)g+h\right)\right]\left[g+(x-1)h\right]} \tag{4}$$

Now, consider the following function:

$$F(\theta^{**},\theta^{**}) = \left(\alpha(\theta^{**},\theta^{**}) + \beta(\theta^{**},\theta^{**})\right) - \left(1 - \beta(\underline{\theta},\bar{\theta})\right).$$

$F(\bullet)$ corresponds to the difference between the probability of breaking a tie during the second half period and the probability that a team leading at half time loses its advantage. Note that it is increasing in θ**, the degree of offensive play in the second half when the match is tied by half time. We are now in a position to compare the strategies of teams under different scoring rules.[8]

Proposition 1: The 3PV rule.

(a) θ*(2) = θ**(2): Under 2PV and conditional on the match being tied, teams do not change their strategy between the first and the second half.

(b) $\theta^{**}(x) > \theta^*(x)$ for all $x > 2$: Under 3PV and conditional on the match being tied, teams always play more offensively in the second half than in the first half.

(c) If $F(\theta^{**}, \theta^{**}) > 0$, then $\partial\theta^*(x)/\partial x < 0 < \partial\theta^{**}(x)/\partial x$: Under 3PV and conditional on the match being tied, teams play more defensively in the first half and more offensively in the second half than under 2PV.

Proof. Part a is immediate if we set $x = 2$ in Equations 1 and 3. Given that $\alpha_2(\theta)/\alpha_1(\theta)$ is increasing in θ and using Equations 1 and 3, we have

$$\theta^{**}(x) \gtreqless \theta^*(x) \Leftrightarrow \frac{\alpha_2(\theta^{**})}{\alpha_1(\theta^{**})} \gtreqless \frac{\alpha_2(\theta^*)}{\alpha_1(\theta^*)} \Leftrightarrow \frac{h[(x-1)^2-1]}{g+(x-1)h} \gtreqless 0$$

Therefore, $\theta^{**}(x) > \theta^{*}(x)$ for all $x > 2$ (part b). Last, from Equation 4, we deduce that $g < h$, or equivalently, $F(\theta^{**}, \theta^{**}) > 0$, is a sufficient condition for $\partial\theta^{*}(x)/\partial x < 0$ (part c). □

The idea behind parts a and b rests on a standard option value argument. A team that concedes an early goal may still tie or win the match, and a team that scores an early goal may still tie or lose the match. Naturally, the absolute change in pay-off from victory to defeat and from defeat to victory is symmetric ($|x|$). The crucial issue when $x > 2$ is that the absolute change in pay-off between a tie and a defeat is smaller than the absolute change in pay-off between a tie and a victory ($|1| < |x - 1|$). As a result, for $x > 2$, the benefits of scoring a goal early in the match (in terms of the increase in the expected final pay-off) are smaller than the costs of conceding an early goal (in terms of the decrease in the expected final pay-off). Stated differently, a team that receives an early goal can mostly hope to move from 0 to 1 point, whereas a team that scores an early goal still has chances of moving from x to 1 point. Because $x - 1 > 1$, the possible loss after scoring an early goal is greater than the possible benefit after receiving a goal. Hence, teams prefer to play relatively more defensively at the beginning of the match so as to avoid conceding an early goal, even if it comes at the expense of also decreasing the chances of scoring. When $x = 2$, leading and being led are symmetric events in terms of expected pay-offs. In that case, teams do not modify their strategy over time as long as the match is tied.

Building on that argument, part c shows that increasing the reward of a victory may have the perverse effect of increasing the incentives of teams to play defensively during the first half period ($\partial\theta^{*} / \partial x < 0$). Indeed, suppose that the probability of breaking a tie during the second half, $\alpha(\theta^{**}, \theta^{**}) + \beta(\theta^{**}, \theta^{**})$, is greater than the probability that a team leading in the score at half time does not win the match, $1 - \beta(\underline{\theta}, \overline{\theta})$. Technically, this corresponds to $F(\theta^{**}, \theta^{**}) > 0$. In this case, the optimal strategy of teams is to play very defensively at the beginning of the game (so as to avoid being led early in the match) and very offensively toward the end (so as to break the tie in one direction or another late in the match). Note that a higher reward for a victory translates into a higher degree of offensive play during the second half ($\partial\theta^{**} / \partial x > 0$), which itself implies a higher likelihood of defensive play during the first half ($\partial F / \partial\theta^{**} > 0$).

Overall, a static reasoning suggests that increasing the pay-off x of a victory always increases the incentives of teams to play offensively (see Equation 2). However, this conclusion may not necessarily remain valid as soon as we account for the dynamic nature of the game and the possibility of changing strategies over time (see the numerical example in the next section). It is interesting that the usual argument against an excessively high pay-off x builds on a fairness consideration: Because soccer is an inherently stochastic game, high distortions in the pay-off of a victory may reward luck in excess. Our model argues that the optimal reward for a victory can be bounded above even if we consider exclusively the incentives of risk-neutral teams to play offensively.

Our model could be extended in a number of directions. First, we could include some strategic interactions, such as a complementarity between the level of offensive play of a team and the marginal probability of scoring of its rival ($\alpha_{12}[\theta^A, \theta^B] > 0$). Second, teams can realistically score two more goals than their opponent in a given half period, in which case a team leading at half time may still end up losing the game. Third and more important, if dynamic considerations are key, then choosing a strategy only twice during the match is still too simplistic. One may wonder what would be the equilibrium if teams can change their strategy as often as they wish. Our simple model cannot answer these questions.

An empirical test of our theory is, although interesting, out of the scope of this article. However, we may obtain some insights from previous empirical analyses. Palomino et al. (1999) suggested that under 3PV, more goals are scored toward the end of the matches than toward the beginning. This seems consistent with the idea that under 3PV, teams adopt more defensive strategies early in the game and more offensive strategies late in the game. Palacios-Huerta (1999) showed that the 3PV rule has not affected significantly the average number of goals in the English Premier League, which seems to indicate that the average level of offensive play is similar under 2PV than under 3PV.[9]

A Simple Numerical Example

To illustrate the idea that the average strategy under 3PV may be more defensive than under 2PV, consider the following stylized functions that satisfy Assumptions 1, 2, and 3: $\alpha(\theta_A, \theta_B) = k\theta_A + l\theta_B^2/2$ and by symmetry, $\beta(\theta_A, \theta_B) = l\theta_A^2/2 + k\theta_B$, with $(\theta_A, \theta_B) \in [0, 1]^2$. This means that k represents each team's marginal benefit of playing offensively (i.e., the increase in the probability of scoring) and $l\theta$ represents the marginal cost (i.e., the increase in the probability of conceding a goal). Using Equations 1 and 3, it can be easily checked that $\theta^*(2) = \theta^{**}(2) = k/l$. Also, $\theta^{**}(3) = 2k/l$, $\alpha\,[\theta^{**}(3), \theta^{**}(3)] = 4k^2/l$, and $\beta(\underline{\theta}, \overline{\theta}) = k$. We also have the following:

$$\theta^*(3) = \frac{k}{l}\left[\frac{2l(1-k) - 4k^2}{l(1-k) + 4k^2}\right] \Rightarrow \frac{\theta^*(3) + \theta^{**}(3)}{2} = \frac{k}{l}\left[\frac{2l(1-k) + 2k^2}{l(1-k) + 4k^2}\right].$$

We can finally compare the average degree of offensive play under 2PV and 3PV as follows:

$$\frac{\theta^*(3) + \theta^{**}(3)}{2} \gtreqless \frac{\theta^*(2) + \theta^{**}(2)}{2} \Leftrightarrow l \gtreqless 2k^2/(1-k).$$

Overall, as the marginal benefit of playing offensively increases (k increases) or its marginal cost decreases (l decreases), the benefits of a constant strategy during the whole match (2PV) outweigh the benefits of a defensive strategy in the first half and an offensive strategy in the second half (3PV).

A Model of the GG Rule

With a very simple extension of our framework, it may be possible to analyze the effect of the GG rule in the strategy of teams. The GG rule has been recently used in the World Cup, the European Cup, and other tournaments at the elimination stage when one and only one team must advance to the next round (it has never been used in pool matches). Before the introduction of this rule, two teams finishing the match tied played during a fixed 30-minute overtime. If the draw persisted, the winner was selected by penalty kicks. According to the new GG rule, the first team to score within the 30 minutes of extra time wins the match. If there are no goals after 30 minutes, then the winner is again determined with the penalty kick method.

The GG rule has two obvious effects: It reduces the expected time of play, and other things being equal, it decreases the probability of deciding the winner by penalty kicks. However, one may wonder if teams adopt more offensive strategies under the GG rule or under the traditional system. In fact, this is important not only because maximizing the level of offensive play is part of the objective function but also because it determines whether in equilibrium fewer matches reach the penalty kick stage (which is the other objective of the rule).

To answer this question, consider the following extension of the model presented in the previous section. The regular game has finished with a draw, and we now model the overtime and only the overtime. Following the previous notations, call $t \in \{1, 2, 3\}$ the beginning, half time, and end of the overtime and consider two different possibilities. In Scenario 1, teams play the entire extra time, and only if it ends up with a draw do they proceed to the penalty kick. In Scenario 2, teams play under the GG rule. The main property of the GG rule is captured with the following assumption: If by half time ($t = 2$), one team has scored one more goal than its rival, it is declared the winner. Otherwise, they play the second half. If by the end of the extra time, the draw still persists, then teams proceed to the penalty kicks.[10]

Given that both teams are equally strong, it seems reasonable to assume that each of them will win at penalty kicks with a probability of 1/2. Therefore, there is no loss of generality by normalizing the pay-off of a victory to 2, the pay-off of a draw to 1, and the pay-off of a defeat to 0. Scenario 1 (the traditional system) is then formally equivalent to the analysis of the previous section when $x = 2$. We are therefore left to analyze Scenario 2. Denote by $\{\gamma_t^i, \gamma^*, \gamma^{**}\}$ the analogue of $\{\theta_t^i, \theta^*, \theta^{**}\}$ to our current analysis under the GG rule. By definition of GG, if either $e_1 = a$ or $e_1 = b$ is realized during the first half period of the extra time, then the game is over. If the match is tied by half time, then it continues for the second half period. Team A's value function at this point is as follows:

$$v_2^A(\gamma_2^A, \gamma_2^B | o) = \alpha(\gamma_2^A, \gamma_2^B)2 + \left[1 - \alpha(\gamma_2^A, \gamma_2^B) - \beta(\gamma_2^A, \gamma_2^B)\right].$$

The equilibrium strategy γ^{**} selected by both teams for the second half period in case of a tie at half time is unique and solves the following:

$$\frac{\alpha_2(\gamma^{**})}{\alpha_1(\gamma^{**})} = 1. \tag{5}$$

By backward induction and once we know the equilibrium of the subgame that starts at $t = 2$, we can determine the behavior of players at the beginning of the overtime. Naturally, at that point, the game is tied. Team A's value function is then as follows:

$$v_1^A(\gamma_1^A, \gamma_1^B) = \alpha(\gamma_1^A, \gamma_1^B)2 + \left[1 - \alpha(\gamma_1^A, \gamma_1^B) - \beta(\gamma_1^A, \gamma_1^B)\right] v_2^A(\gamma^{**}, \gamma^{**} | o),$$

where compared with $v_1^A(\theta_1^A, \theta_1^B)$, we have simply replaced $v_2^A(\underline{\theta}, \overline{\theta} \mid a)$, $v_2^A(\theta^{**}, \theta^{**} | o)$ and $v_2^A(\underline{\theta}, \overline{\theta} \mid b)$ by 2, $v_2^A(\gamma^{**}, \gamma^{**} \mid o)$ and 0, respectively. The equilibrium strategies γ^* of both teams for the first half period are the analogue of the strategies in Equation 3, that is,

$$\frac{\alpha_2(\gamma^*)}{\alpha_1(\gamma^*)} = \frac{2 - v_2^A(\gamma^{**}, \gamma^{**} | o)}{v_2^A(\gamma^{**}, \gamma^{**} | o)} \equiv \frac{1 - \alpha(\gamma^{**}, \gamma^{**}) + \beta(\gamma^{**}, \gamma^{**})}{1 + \alpha(\gamma^{**}, \gamma^{**}) - \beta(\gamma^{**}, \gamma^{**})} = 1, \tag{6}$$

and we can state our second result.

Proposition 2: The GG rule. $\gamma^* = \gamma^{**} = \theta^*(2) = \theta^{**}(2)$: In elimination tournaments, the GG rule does not affect the incentives of teams to play offensively.

Proof. The proof is immediate by comparing Equations 5 and 6 with Equations 1 and 3 when $x = 2$.

In elimination tournaments, adopting the GG rule increases the variance in the pay-off of playing offensively. Indeed, once team A has scored a goal (event $e_1 = a$), its rival B does not have the opportunity to come back in the game. That is, team A gets a pay-off of 2 $(> v_2^A(\bullet | a))$ and team B a pay-off of 0 $(< v_2^B(\bullet | a))$. However, the increase in the variance of pay-offs is symmetric. Formally,

$$2 - v_2^A(\underline{\theta}, \overline{\theta} \mid a) = \beta(\underline{\theta}, \overline{\theta}) = v_2^B(\underline{\theta}, \overline{\theta} \mid a) - 0.$$

As a result and exactly for the same reasons as in Proposition 1(a), the incentives of teams to attack are the same in Scenario 1 (traditional system) and Scenario 2

(GG rule). Overall, Proposition 2 shows that the GG rule fulfills its mission: Because teams play as offensively as under the previous system, the equilibrium probability of reaching the (unsatisfactory) penalty kick phase is reduced.

A POSITIVE ANALYSIS OF 3PV AND GG

Given the lessons learned with Propositions 1 and 2, one could consider the possibility of designing a simple combination of the 3PV and GG rules in league tournaments. In fact, this would be relatively easy to implement. For example, instead of a fixed 90 minutes of play, a regular match could last only 70 minutes. In case of a draw after regular time, teams would play for an extra 20 minutes under the GG rule.

What would be the effect of such a combination? To answer this question, we formalize once again our argument with a simple extension of the basic league tournament model presented in Section 2. As usual, there are three dates $t \in \{1, 2, 3\}$. At $t = 1$ (Minute 0), the game starts and teams choose their strategy. At $t = 2$ (e.g., Minute 70), the regular game ends. If one team has scored more goals than its opponent, the game is over. The winner obtains x points and the loser obtains 0 points. If on the other hand, the game is tied, then teams keep playing for a fixed extra period. At $t =$ 3 (e.g., Minute 90), the match is stopped independently of the score, with x points being allocated to the winner, 0 points to the loser, and 1 point to each team in the case of a draw.[11] We can then compare the strategy of teams in this scenario (which we call 3 + G) with the scenario in which the game is never stopped at $t = 2$. In fact, the latter case corresponds to the 3PV model analyzed in Section 2, with equilibrium strategies $\theta^*(x)$ and $\theta^{**}(x)$.[12]

Denote by $\{\mu_t^i, \mu^*, \mu^{**}\}$ the analogue of $\{\theta_t^i, \theta^*, \theta^{**}\}$ to our current analysis under the 3 + G. If the match is tied by the end of the regular time, we proceed to the extra 20 minutes of play. Team A's value function at that point is as follows:

$$v_2^A(\mu_2^A, \mu_2^B | o) = \alpha(\mu_2^A, \mu_2^B)x + \left[1 - \alpha(\mu_2^A, \mu_2^B) - \beta(\mu_2^A, \mu_2^B)\right].$$

Therefore, the equilibrium strategy μ^{**} adopted by both teams for the extra 20 minutes of play (between $t = 2$ and $t = 3$) is unique and solves the following:

$$\frac{\alpha_2(\mu^{**})}{\alpha_1(\mu^{**})} = x - 1 \tag{7}$$

At the beginning of the match, the game is tied. Also, teams anticipate that the game will stop at $t = 2$ if at that point, one team has scored more goals than the rival, with the pay-offs being x for the winner and 0 for the loser. Only if the match remains tied will teams proceed to extra time. Therefore, team A's value function at the beginning of the match is as follows:

$$v_1^A(\mu_1^A, \mu_1^B) = \alpha(\mu_1^A, \mu_1^B)x + \left[1 - \alpha(\mu_1^A, \mu_1^B) - \beta(\mu_1^A, \mu_1^B)\right] v_2^A(\mu^{**}, \mu^{**} | o)$$

Thus, the equilibrium strategy μ^* adopted by both teams for the regular game is unique and solves the following:

$$\frac{\alpha_2(\mu^*)}{\alpha_1(\mu^*)} = \frac{x - v_2^A(\mu^{**}, \mu^{**}|o)}{v_2^A(\mu^{**}, \mu^{**}|o)} \equiv \frac{(x-1)[1-\alpha(\mu^{**}, \mu^{**})] + \beta(\mu^{**}, \mu^{**})}{1 - \alpha(\mu^{**}, \mu^{**}) + (x-1)\beta(\mu^{**}, \mu^{**})}. \tag{8}$$

In fact, the only difference between Equations 7 and 8 and Equations 5 and 6 is that in elimination tournaments, the pay-off of a tie is necessary half way between the pay-off of a victory and the pay-off of a defeat because teams proceed to the penalty kick. So by definition, $x = 2$. By contrast, in league tournaments, it is still possible to have a GG-type rule (the game is stopped if and only if one team is winning) and yet keep the possibility of distorting the pay-off of a victory (i.e., final pay-offs of [x, 1, 0]). We are therefore in a position to provide a recommendation for a change in the rules.

Proposition 3: The 3 + G rule. $\mu^*(x) > \theta^*(x)$ and $\mu^{**}(x) = \theta^{**}(x)$ for all $x > 2$: In league tournaments, the average incentives of teams to play offensively is always higher under 3 + G than under 3PV alone.

Proof. $\mu^{**}(x) = \theta^{**}(x)$ is immediate given Equations 1 and 7. Also, given Equations 3 and 8, we have $\mu^*(x) > \underline{\theta}^*(x) \Leftrightarrow [(x-1)^2 - 1]\,[(1 - \alpha\,(\mu^{**}, \mu^{**})] > [(x-1)^2 - 1]\,[1 - \alpha\,(\theta^{**}, \theta^{**}) - \beta(\underline{\theta}, \overline{\theta})]$, and the result follows. □

The greatest benefits of GG are best highlighted when we combine this rule with the 3PV system. Proposition 3 shows that an increase in the variance of pay-offs (obtained via the GG rule) together with an increase in the expected benefit of breaking a tie (obtained via the 3PV rule) encourages teams to adopt more offensive strategies toward the beginning of the match without affecting their incentives toward the end of it. This is simply because the GG rule transforms the early stage of a game into a potential final stage. In other words, the combination of 3PV and GG increases the expected benefit of adopting a risky strategy early in the match relative to its cost. Formally,

$$x - v_2^A(\mu^{**}, \mu^{**}|a) \equiv \beta\,(\mu^{**}, \mu^{**})(x-1) > \beta(\mu^{**}, \mu^{**}) \equiv v_2^B(\mu^{**}, \mu^{**}|a) - 0 \quad \forall x > 2.$$

To sum up, it is clear that sport rules are necessarily part of a second-best world. However, our article suggests that well-constructed models can capture the main underlying effects of a rule. With their help, it is possible to evaluate the suitability of past changes and to design new rules that are simple and easy to implement and that constitute an improvement over the existing ones.

CONCLUSION

Soccer is by far the most popular sport throughout the world. Events such as the World Cup or the European Champions League capture the attention of millions of supporters. It has been argued that money and excessive coverage by the media has corrupted the game. Stakes have become so high that spectacular matches are now the exception rather than the rule even if, nobody doubts, the average quality of players rises constantly. Several measures have been adopted in order to keep the thrill of the game. However, to our knowledge, only rules of thumb rather than carefully thought arguments have been presented as the main reasons in favor of the changes proposed.

This article has demonstrated that simple economic concepts (such as dynamic optimization, option value, and Nash equilibrium) and a rigorous modeling can be helpful in understanding some basic (although nontrivial) effects of sport rules on the behavior of players. Using these elements, we have studied the merits of two controversial changes in soccer rules: the 3PV and the GG. From a positive viewpoint, our analysis has drawn simple and clear recommendations for the modification of rules. Given the availability of data on soccer, a natural next step would be to test our predictions.

Last, one might wonder whether the design of sport rules deserves the attention of researchers. In our view, professional sport has become an important part of our everyday lives. Many decisions, including allocation of leisure, allocation of money, and social conduct, are affected by sport events. We therefore feel that there is indeed an economic and social interest in understanding sports and optimizing their rules.

NOTES

1. It is different to compare two-point victory (2PV) versus three-point victory (3PV) (i.e., $x = 2$ versus $x = 3$) in a league of four teams than in a league of 18 teams. The objectives also play an important role (e.g., to win the league tournament versus to be among the top two teams in the pool so as to advance into the next round). These considerations can be included just by rescaling the value of a victory.

2. A more realistic modeling would allow teams to modify their strategy continuously (and especially whenever they score or concede a goal).

3. We attract the attention of the reader to the fact that the assumption is not that the probabilities of scoring and conceding one goal are increasing in the level of attack. The distinction makes sense only because strategies are chosen at $t = 1$ and $t = 2$ rather than at every single point in time. Note also that if the score changes during a half period, teams are most likely to modify their strategy within that half period. Our three-period model does not capture this property (see also note 1).

4. Assumption 2 implies that the functions α_i, α_{ii}, β_i, and β_{ii} depend only on one argument. Also, given Assumption 3, $\alpha_1(\theta') = \beta_2(\theta')$, $\alpha_2(\theta') = \beta_1(\theta')$, $\alpha_{11}(\theta') = \beta_{22}(\theta')$, and $\alpha_{22}(\theta') = \beta_{11}(\theta')$. Last, if both teams play the same strategy, they have the same chances of winning and losing: $\alpha(\theta', \theta') = \beta(\theta', \theta')$.

5. Palomino, Rigotti, and Rustichini (1999) discussed extensively the controversy among soccer fans on whether the cross-derivative is more likely to be positive or negative.

6. Note that Assumption 2 is responsible for the uniqueness of θ^{**}: Team *A*'s marginal benefit and marginal cost of playing a given strategy, $\alpha_1(\bullet)$ and $\beta_1(\bullet)$, are independent of the equilibrium strategy played by team *B* and vice versa.

7. The fact that teams play the most offensive and the most defensive strategy when they are losing and winning, respectively, is excessively simplistic. However, it is not a necessary ingredient for our analysis. Also, given $\alpha_{11} \le 0$, $\alpha_{22} > 0$, $\alpha_{12} = 0$, it is immediate to check that the second-order condition is satisfied globally:

$$\frac{\partial^2 v_2^A(\theta, \theta_2^B \mid o)}{\partial\theta^2} = (x-1)\alpha_{11}(\theta) - \alpha_{22}(\theta) < 0 \text{ for all } \theta\text{; therefore } \theta^{**} \text{ is indeed a maximum.}$$

8. The technical result of Proposition 1 parts b and c is given for all $x \ge 2$. However, for expositional purposes, the interpretation is given in terms of 3PV versus 2PV ($x = 3$ versus $x = 2$).

9. We should be cautious when interpreting these results. First, Palomino etal. (1999) do not analyze the evolution of the number of goals under 2PV. Second, our 3PV theory predicts an increase in the probability of scoring as time elapses but only conditional on the game being tied.

10. Naturally, this modeling captures only imperfectly the effects of the GG rule. It is interesting that the Union Européenne de Football–Association has recently changed of the rule for the 2003 Champions League (the championship where the top European clubs meet). In the new version of GG, the match is not stopped once a team scores but only after 15 minutes of overtime if one team is leading. If no team is winning at that point, then they play for the full 30 minutes. Our modeling of GG is therefore closer to this new rule (for the details, see http://uefa.com/Competitions/UCL/news/kind=1/newsID=22214.html).

11. The National Hockey League (NHL) has a similar rule for the regular season: In case of a draw, teams play an overtime under the GG rule and, if no team scores in the overtime, they both get 1 point. However, the value of a victory is always two points. This means that GG is adopted in regular season games but is not combined with 3PV.

12. Again, it would be more realistic to allow a finer partition of time and more possibilities of modifying the strategy. Also, the reader might be concerned with the fact that the expected time of play under 3 + G can be substantially smaller than under 3PV. One could then compare, for example, 3PV with a fixed 90 minutes of play with 3 + G with 80 minutes of play and 20 minutes extra in case of a tie.

REFERENCES

Chan, W., Courty, P., & Hao, L. (2001). *Suspense* [Mimeo]. London: London Business School.

Chiappori, P. A., Levitt, S., & Groseclose, T. (2002). Testing mixed strategy equilibria when players are heterogeneous: The case of penalty kicks in soccer. *American Economic Review*, *92*, 1138-1151.

Lazear, E. P., & Rosen, S. (1981). Rank-order tournaments as optimum labor contracts. *Journal of Political Economy*, *89*, 841-864.

Palacios-Huerta, I. (1999). *One century of the world's most popular sport* [Mimeo]. Providence, RI: Brown University.

Palacios-Huerta, I. (in press). Professionals play minimax. *Review of Economic Studies*.

Palomino, F., Rigotti, L., & Rustichini, A. (1999). *Skill, strategy and passion: An empirical analysis of soccer* [Mimeo]. Tilburg University.

Walker, M., & Wooders, J. (2001). Minimax play at Wimbledon. *American Economic Review*, *91*, 1521-1538.

Isabelle Brocas is an assistant professor of economics at the University of Southern California and a research affiliate at the Center of Economic Policy Research. Her work on the economics of research and development characterizes efficient policies to provide incentives to innovate. She also works on behavioral economics, auction theory, and sports economics.

Juan D. Carrillo is an associate professor of economics at the University of Southern California and a research affiliate at the Center of Economic Policy Research. His work on behavioral economics analyzes self-perception and decision making under time-inconsistent preferences. He has also worked on contract theory, political economics, and sport economics.