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How long is a minute? ☆

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ABSTRACT

Psychophysics studies suggest that our perception of time is different from the objective passage of time. Economics research emphasizes that the value of a reward depends on the delay involved. In this paper, we combine both strands and estimate *time perception* and *time discounting* functions at the individual level in an incentivized controlled laboratory environment. We find a negative and statistically significant correlation between time perception and time discounting: subjects who overestimate objective time intervals are less willing to delay gratification. The result suggests that our ability to delay consumption is related to our mental representation of time delays.

> Time is too slow for those who wait, too swift for those who fear, too long for those who grieve, too short for those who rejoice, but for those who love, time is not.
>
> [Henry Van Dyke ("Time Is" – Music and Other Poems, 1904)]

1. Introduction

As the poet elegantly expressed, time is subjective. It flies when you enjoy and virtually stops when you suffer. Tomorrow is "in a very long time" for kids and "practically now" for seniors. While the feeling of the passage of time is not clearly defined, the observation suggests an intriguing possibility, namely a connection between how we perceive time and how we discount time. The goal of this paper is to formally explore such relationship in a narrow but well-defined objective setting. Our conjecture is simple: if one person perceives one week to be longer than another person, it seems natural that he will be less willing to delay a reward by that amount of time. Even though inter-temporal decisions likely depend on many different cognitive processes, we hypothesize that timekeeping mechanisms are partly responsible for observed choices. If the hypoth-

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esis is correct, it can help understand the paradoxical tendency of older adults to save more than younger adults (Banks et al., 1998) despite their shorter life expectancy. More generally, validating our hypothesis would suggest that eliciting discount rates is a valuable but incomplete measure to understand the intertemporal tradeoffs that different people make.

To address this question we ask subjects to perform two tasks in a controlled laboratory environment. First, we elicit their *time discount rates* using the method proposed by Andreoni and Sprenger (2012a) (hereafter, [AS]), where subjects allocate a fixed amount of tokens between two dates. We use their convex time budget (CTB) method due to its robustness, and structurally estimate a quasi-hyperbolic discount function and curvature of utility.[1] Second, we elicit their *time perception estimates* using a task adapted from the psychophysics literature and extended in several ways, where subjects reproduce intervals of lengths ranging between 20 seconds and 4 minutes. Formally, we ask them to click the start box to begin a time interval and click again when a predetermined amount of time (e.g., 2 minutes and 31 seconds) has passed. This task is performed in conjunction with a distractor task that prevents them from counting seconds. We estimate for each individual a time perception power function that maps true time intervals into perceived time intervals. Finally, we correlate the impatience or preference for the present derived from the time discounting task ($\mathcal{TD}$) with the subjective evaluation of time obtained from the time perception task ($\mathcal{TP}$).

We find substantial dispersion in the time discounting of our subjects. The estimated parameters in the $\mathcal{TD}$ task are in line with those found in [AS], with low levels of impatience, little evidence of present bias and some small (but positive) concavity in the utility function. Perception estimates in the $\mathcal{TP}$ task are also heterogeneous. Although a large fraction of individuals systematically underestimate time (around 40%), we also observe the opposite tendency in a number of subjects (around 30%). More generally, we find evidence of both concave and convex time perception functions.

The main novelty of the study is to analyze the relationship between time perception and time discounting. To this purpose, we correlate the estimated perception function with the estimated discount function. We first show that our subjects can be ranked consistently in their time perception and time discounting attitude for delays in the range of 1 hour to 1 week. We can then perform a correlation analysis at the individual level within this time range. For all intervals, we find a statistically significant negative correlation between the level of impatience estimated in the $\mathcal{TD}$ task and the perception of time estimated in the $\mathcal{TP}$ task: the Pearson correlation coefficient (PCC) ranges between -0.20 and -0.30 with a p-value between 0.01 and 0.05 depending on the delays, conditions and functional specifications. In other words and consistent with our hypothesis, subjects who overestimate objective time intervals are less likely to delay consumption by that amount of time than subjects who under-estimate them. Said differently, timekeeping mechanisms are related to inter-temporal decision-making.

We then use this result to build a simple model of discounting based on time perception. Formally, we assume that subjects mentally represent the perceived time of a given true delay and apply a discount to this perceived delay. We find that the fit of this model is on aggregate equally good than that of the original model (though not strictly better), suggesting that time perception is a likely driver of mental discount computations when we evaluate future rewards.

Our paper relates to the growing literatures on time discounting and time perception. Time discounting has received much attention in economics. Researchers have proposed different parametric formulations of the time discounting function as well as different experimental designs to elicit them, and the empirical and experimental estimates vary significantly across studies. There are two main challenges for the elicitation of time discounting. First, subjects may not be time-consistent and overweigh immediate gratification relative to all future ones. This has motivated hyperbolic specifications of time discounting as opposed to the standard exponential formulation.[2] Second, time is inherently uncertain and deciding to postpone consumption amounts to choosing a lottery. It is therefore important to be able to disentangle risk preferences from time preferences. Indeed, when choosing between consumption now and consumption in the future, a subject may choose the former because uncertainty about the future makes the present option more desirable. The recent literature proposes methods to jointly estimate time and risk preferences (Andersen et al., 2008; Andreoni and Sprenger, 2012a, 2012b; Andersen et al., 2014) and reports less or no evidence of a present bias. Our analysis relies on this last line of research, which allows us to better isolate time discounting.

Time perception has also been extensively studied in the psychophysics literature. It is centered on prospective time evaluations, where subjects are informed beforehand that they have to make a time related judgment. These studies mostly focus on extremely short intervals (milliseconds, seconds) and use non-incentivized methods in which subjects have either to verbally assess a duration, reproduce or produce a time interval, or compare the duration of intervals presented successively (Lorraine, 1979; Grondin, 2010). There are two major findings in this literature. First, prospective time evaluation is often consistent with Weber–Fechner's law of human perception, implying that subjective time is not linear in true time but rather proportional to its logarithm (Grondin, 2001). Second, individual evaluations are qualitatively similar for time intervals of different lengths (Lewis and Miall, 2009), suggesting the existence of a single 'internal clock' mechanism that

[1] One advantage of CTB is that it controls for diminishing marginal utility. It has been recently employed in a wide variety on contexts (see e.g., Andreoni et al., 2015; Augenblick et al., 2015; Carvalho et al., 2016, and Kuhn et al., 2015). However, it has also received some criticisms (Harrison et al., 2013). We realize that different methods have different advantages and use one which has proved simple and reliable. The paper does not take a stand on the debate over the advantages of different methods and does not attempt to improve upon them.

[2] The quasi hyperbolic formulation was first developed by Phelps and Pollak (1968) in a model of imperfect intergenerational altruism. It was later reintroduced by Laibson (1997) to study the dynamic choices of an individual who overweighs the present. Loewenstein and Prelec (1992) propose a general hyperbolic specification.

governs prospective timing. Our study draws on the concept developed in this literature and also focuses on prospective timing. However, we propose a new methodology that departs from the existing literature in several important respects. First, we focus on significantly longer time intervals than the majority of the literature (several minutes). Second, we introduce a new and incentivized elicitation method paired with a distractor task that prevents subjects from counting. Third, we provide a structural estimation of a two-parameter power function of time perception instead of imposing a logarithmic form. Indeed, although there is evidence of a concave relationship between true and perceived time for many subjects, there is also a significant fraction of individuals for which the opposite, convex relationship fits better. Finally, we estimate the perception function at the individual – not the aggregate – level. This allows us to study heterogeneity in perception across subjects.[3] Despite these methodological differences, our study builds on the existing knowledge and paradigms developed in psychophysics, and we also borrow the terminology of the field. In particular, the reports provided by subjects will be labeled as "perceived time" or "subjective time" measurements.

Finally, we are not the first to argue the possibility of a relationship between time perception and time discounting. This has been theorized in several studies. Read (2001) tested for sub-additivity in time discounting and suggested that a possible reason for this effect was the subjective evaluation of time (due for example to differences in memory and attention). Lemoine (2015) built a model that captures the acceleration of time as we age and Capra and Park (2015) studied the effect of time distortions. However, most of the literature has focused on the potential relationship between time perception and present bias. In particular, Ray and Bossaerts (2011) proposed a theoretical approach to show that choices are present-biased with respect to calendar time if individuals discount the future exponentially with respect to biological time while the internal representation of time is stochastic and autocorrelated. Cui (2011) demonstrated that the scalar property of time perception also implies hyperbolic discounting.[4] A few studies have used experimental techniques to relate present-bias and impulsivity to time perception (Zauberman et al., 2009 and Han and Takahashi, 2012). Their aggregate analysis reveals hyperbolic discounting with respect to objective time but exponential discounting with respect to subjective time.[5] In the present study, we are interested in a more fundamental question: we want to test at the individual level whether subjects who perceive objective time as subjectively longer will be less prone to delay consumption. We are not directly interested in time discounting biases and there is, in fact, little evidence in our data in favor of hyperbolic discounting.

2. The experiment

2.1. Design and procedures

The experiment was conducted in the Los Angeles Behavioral Economics Laboratory (LABEL) at the University of Southern California.[6] A total of 124 subjects participated in the study in 14 groups of 6 to 10 participants each. In order to participate in the experiment, subjects were required to be enrolled USC students with a USC Discretionary Card Account. Students frequently use their USC Card to pay in businesses on campus and the surrounding area. By special arrangement with the USC Card Department, we were able to deposit money into their accounts at our convenience.

Sessions lasted for about 1 h 30 min and started either at 10 am or 12 pm. They consisted of two main tasks, always administered in the same order: time discounting task followed by time perception task. Instructions were read out loud at the beginning of each task. Since our time perception task is the most novel of the two, we will all along the paper discuss it first (but remember that it was administered second).[7] The complete instructions are presented in a separate "General Instructions" online appendix.

Time perception task. Participants were asked to produce 9 time intervals τ of 24, 31, 41, 53, 69, 89, 116, 151 and 196 seconds respectively, without knowing in advance the number or length of intervals to produce.[8] We designed a Matlab-based program to implement the elicitation of the participants' time perception. It presented the instructions on the screen and guided subjects to estimate time intervals. Subjects were prompted the length of the interval τ to be estimated. Then, subjects marked the beginning and end of the interval by clicking on a button on the top right corner of the screen. The

[3] Some studies investigate instead retrospective time evaluation, where subjects are not informed beforehand that they will have to make a time related judgment (Block and Zakay, 1997). By definition, under this approach only one measure can be obtained per individual. The studies find that retrospective time evaluations are usually shorter than prospective time evaluations (Fraisse, 1984) and they draw on different (memory) processes. We performed a one-shot retrospective time evaluation task and also found more underestimation than under prospective time evaluation (see Appendix A2).

[4] Takahashi (2005) built a model of dynamic inconsistency based on Weber's law. Wittmann and Paulus (2008) proposed a model that relates impulsivity to time experience.

[5] These studies were not incentivized and produced unreasonably high discount rates (e.g., 160% annual rate for three-month delays). Importantly, they did not use a conventional time perception task either. Instead, time perception was measured by marking how long future delays (three months, one year, three years) felt on a line scale. In our view, such approach is unsuitable to elicit time perception estimates.

[6] For information about the lab, please visit http://dornsife.usc.edu/label.

[7] We chose that order because we thought that actions in the time perception task could prime choices in the time discounting task. By contrast, it seemed less plausible that choices in the time discounting task would impact the accuracy of subjects in the time perception task.

[8] This is called a *prospective* time estimate in a *production* paradigm. Prospective (as opposed to retrospective) refers to a case where subjects know in advance that they will be requested to estimate the elapsed time. Production occurs when subject are informed about the length of the interval they must produce (Nichelli, 1996). This is different from reproduction, where subjects experience a time interval (without knowing its real length) and are then asked to reproduce a second interval of the same size.

Fig. 1. Example of a filler task.

order for the 9 intervals was randomly selected but it was the same for all subjects.[9] The reports collected will be referred to as subjective or perceived time.

To make sure that subjects did not count time, we asked them to solve novel filler (distracting) tasks while estimating time intervals.[10] In each of these tasks, subjects were presented a 4×6 table where each row and column had a name and they were instructed to click on a specific cell. In the example of Fig. 1, subjects were asked "Please click the cell where the column to the right of the column labeled athena intersects the row above the biology row".

The names of the rows and columns as well as the phrasing of which cell to click on changed from table to table to make sure subjects would pay attention. There was a random and unspecified time limit to complete each task (between 10 and 15 seconds) and failure to complete it counted as an incorrect answer.[11,12] The amount earned depended on the proportion of correct answers in the filler tasks and the distance between time estimates and true time intervals. For each subject, one time interval was randomly selected for payment. For this time interval, the subject earned money only if at least 80% of the filler tasks were correctly answered. The subject would then earn $25 if the estimate was within ±5% of the real length of the interval, $15 if it was within ±10% and $5 if it was within ±20%. If less than 80% of the filler tasks were correctly answered, the subject did not earn anything no matter how good the estimation of the time interval was. The entire procedure was explained beforehand.[13,14]

Time discounting task. Since the goal of the paper was not to provide an innovative way to elicit time discounting, we followed closely the CTB design and allocation procedure in [AS]. We provided subjects with a budget of experimental tokens to allocate either to a sooner time t or a later time $t+k$, at different token exchange rates. The relative rate at which tokens translated into money determined the gross interest rate, $(1+r)$. The amounts allocated at dates t and $t+k$ were denoted by c_t and c_{t+k} respectively. We implemented a 3×3 design with three sooner payment dates starting from the experiment date ($t \in \{0, 7, 21\}$ in days) and three delay lengths ($k \in \{21, 42, 63\}$ also in days). For each of the 9 pairs of (t, k), there were 5 different budgets and exchange rates for a total of 45 sooner-later token allocation tasks. Dates were selected

[9] Before coming to the laboratory, subjects were asked to put away any time-keeping devices such as watches, music players and cell phones. An experimenter made sure that subjects placed these items in their bag and monitored that they did not use any such device.

[10] Chronometric counting is avoided in the psychophysics literature by resorting to interfering tasks. These usually consist in asking participants to repeat aloud digits presented on a computer screen (Wearden et al., 1997). Given we organized sessions with several participants, it was not possible to use such method.

[11] Subjects were informed that there was a time limit of "a few" seconds. They also knew that this unspecified time limit was not constant over tasks. Finally, they had some practice rounds where they could build an estimate of the approximate time they had to complete the task.

[12] The tasks required sufficient effort to prevent subjects from counting but were easy enough to make sure all subjects could complete them if they paid attention. Participants were informed that if they reported the end of an interval during a task, that task would not count as correct or incorrect. Subjects also estimated a 10th time interval of 219 seconds (always performed last), which was not used for analysis. Here is why: participants clicked to report their time interval estimates and their answers to the filler task. All these clicks could be heard by other participants, who could not disentangle between either types of clicks until almost the end of the experiment. As such, there were 9 relevant intervals during which subjects could hear all other subjects clicking to complete either task, and therefore could not make any inference about the time estimations of their peers. Towards the end of the experiment, in the 10th trial, they could potentially use the *absence* of clicking as a cue that the others had finished their time estimations. This could bias their own last time estimate.

[13] We chose this method because it is more intuitive and easier to explain than the (incentive compatible) quadratic scoring rule. Also, there is evidence that reports vary even among different proper scoring rules (Palfrey and Wang, 2009).

[14] In 84% of the trials, subjects answered correctly at least 80% of the filler tasks and therefore were eligible for payment. Subjects did not receive performance feedback during the filler task to make sure they had incentives to estimate time accurately. After each time interval, they were informed of their performance on the filler tasks but not on the time interval estimation task.

to avoid holidays, vacations and examination dates. To avoid differential weekday effects, t and k were both multiples of 7, so that payments were scheduled to arrive on the same day of the week.

Subjects were given 10 tokens for each of the 45 allocation tasks. Tokens assigned to sooner and later payments had values v_t and v_{t+k}, respectively. Since $v_{t+k}/v_t = (1+r)$ is the gross interest rate over k days, $(1+r)^{1/k}$ is the daily interest rate. Values were never multiples of \$0.05 to avoid gravity point effects. To formally implement choices, we provided paper booklets. Subjects had to circle their preferred token allocation among the eleven possible combinations of tokens sooner vs. token later in each of the 45 tasks. Appendix A1 shows the token rates, standardized daily interest rates and corresponding annual interest rates for all 45 budget sets. It also shows the presentation of the first 5 tasks of the paper booklet, corresponding to $t = 0$ and $k = 21$ (Fig. 8 in Supplementary material).

To avoid in-lab vs out-lab payments at different dates, all sooner and later payments, including those for $t = 0$, were deposited into the subjects' USC Card Accounts by 4 pm on the specified date.[15] Subjects were described the payment method and the arrangement made with the USC Card Department. They were told that they would receive a \$4.64 thank-you payment for participating in two payments, \$2.32 at the sooner and \$2.32 at the later date regardless of their choices, and that all experimental earnings were added to these two payments. Subjects knew in advance that one of the 45 choices was going to be selected for payment by drawing a numbered ball from a bingo cage. They were given Professor Juan Carrillo's business card and they were told to contact him if payments did not reflect in their account, in which case a payment would be hand-delivered immediately. Subjects were asked if they trusted the payment method at the end of the experiment and 95% of respondents replied yes.[16]

Other tasks. We conducted three peripheral tasks: a one-shot retrospective time estimate task, a cognitive ability test, and a survey to collect demographic information. We used the data collected in those tasks as controls in our regression analysis. Further details of the procedures and results obtained in these tasks can be found in Appendix A2.

From the 124 subjects who participated in the study, four subjects were excluded due to data recording issues, leaving us with a sample of 120 subjects.

2.2. Challenges

An experimental study of time perception and time discounting is subject to three challenges. First, the temporal horizons are different. We can realistically elicit multiple prospective time estimates that are on the range of minutes whereas meaningful monetary tradeoffs must involve temporal delays that are on the range of weeks. We will therefore extrapolate our estimates upwards for time perception and downwards for time discounting. Modeling time perception and time discounting will allow us to identify the most reliable extrapolations and we will consider variants to check the robustness of our results.

Second and related, our goal is to compare perception and discounting across individuals. If some time perception functions are not linear in true time and/or some time discount functions are not exponential, rankings may depend on the horizon (for example, a hyperbolic discounting subject may be more impatient in the short run and less impatient in the long run than an exponential discounting subject). In the analysis, we will put special emphasis in determining the time range for which the ranking of the estimates between individuals is preserved.

Third, the mechanisms that govern time perception are physiological whereas time discounting in the context of choices is governed by cognitive processes that involve many interacting systems.[17] It is therefore unlikely to find a perfect correlation between time perception biases and time discounting attitudes. Our objective is simply to reveal any significant relationship between the two processes to help looking into the time discounting black box.

3. Time perception ($\mathcal{TP}$)

We first present the theoretical framework and experimental results of our time perception task. Time perception refers to the fact that an objective length of time may be inaccurately perceived, leading to under- or over-estimation of true delays. This issue has been analyzed as part of the general study of human perception, that is, the relationship between the actual change in a physical stimulus (e.g., the unfolding of objective time) and the perceived change. The original modeling of human perception is known as the Weber–Fechner law (Fechner, 1860) that posits a logarithmic relationship between the physical intensity of a stimulus and its perception. The law has been applied to many sensory areas and, in particular, to time. Psychophysics has documented non-linearities in numerous experiments in which perception is measured

[15] This removes the salience of immediate payment. It is likely to result in the later option being chosen more frequently but it also makes the uncertainty and potential anxiety over payment similar whether it is today or in the future (for a discussion, see Andreoni and Sprenger, 2012a, 2012b).

[16] The full list of differences relative to [AS] are (first item refers to our design, second item to theirs): (i) payment to USC card vs. payment by check; (ii) thank you payments of \$2.32 vs. \$5; (iii) slight differences in (r, t, k, m) but calibrated to equalize daily gross interest rates (see Fig. 8 in Suplementary material); (iv) 11 vs. 101 choices per budget; (v) pen and paper vs. computerized implementation; and (vi) 120 vs. 97 subjects.

[17] Time representation involves mostly the striatum and the basal ganglia (Ivry and Spencer, 2004; Meck, 2005). Temporal choices involve the ventral striatum and several areas of the Prefrontal Cortex (Kable and Glimcher, 2007; Peters and Buechel, 2009). The latter is of significant importance for executive function and attention.

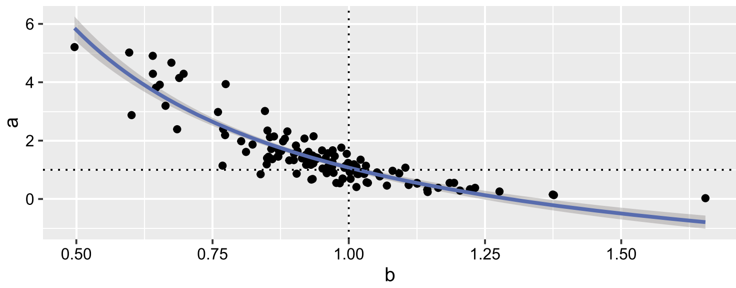

Fig. 2. Individual time perception parameter estimates $(\hat{a}_i, \hat{b}_i)$.

objectively through timing tasks where subjects are asked to produce, reproduce or compare time intervals. The field has also developed theoretical frameworks, such as the scalar timing theory (Gibbon, 1991). Also, research in neuroscience suggests that perceived durations result from the way neurons encode specific durations in their firing rate (Matell and Meck, 2000; Matell et al., 2003). Furthermore, even though most of the experimental timing literature considers perception of short intervals of time (at most a few minutes), animal studies focusing on intervals above one day (Crystal, 2001) as well as time-based prospective memory and time management studies (Francis-Smythe and Robertson, 1999; Waldum and McDaniel, 2016) suggest that a coherent underlying timing mechanism governs time perception and time estimation of future delays over both short and long periods of time.

We consider here a simple model of time perception in which subject i formulates a subjective duration θ_i of a true time interval of length τ according to the function:

$$\theta_i(\tau) = a_i \tau^{b_i} \tag{1}$$

where $a_i = b_i = 1$ corresponds to a correct time perception. This representation is borrowed from Steven's law, a generalization of the Weber–Fechner law, which posits a power relationship between the magnitude of a physical stimulus (distance of an object, brightness of an image, level of a sound, sugar component of a meal, etc.) and its perceived strength (Stevens, 1957). This theoretical relationship captures the non-linearities mentioned above and encompasses perception functions that are concave ($b < 1$), linear ($b = 1$) or convex ($b > 1$) in true time.[18] It has been applied to a variety of problems, including time perception (Stevens, 1975; Luce, 2002).

We fitted this model to the data obtained from the time perception task ($\mathcal{TP}$). For each individual i, we estimated by non-linear least squares (NLS) the parameters a_i and b_i of the following regression:

$$y_{is} = a_i \tau_s^{b_i} + \epsilon_{is} \tag{2}$$

where, for trial $s \in \{1, \ldots, 9\}$, the reported perception of individual i is y_{is}, the true length in seconds is $\tau_s \in \{24, 31, 41, 53, 69, 89, 116, 151, 196\}$, and the noise in the process is $\epsilon_{is} \sim N(0, \sigma_i^2)$. Using a boxplot analysis, we identified three extreme outliers ($\hat{a}_i > 8$).[19] These subjects were excluded from the analysis. Fig. 2 graphically depicts the estimated parameters $(\hat{a}_i, \hat{b}_i)$ of the remaining 117 subjects. For illustrative purposes, Fig. 3 presents the choices of three representative subjects (with $\hat{b}_i > 1$, $\hat{b}_i \simeq 1$ and $\hat{b}_i < 1$, respectively).

We obtained three main findings. First, the power model explains remarkably well the data: the average R^2 is 0.98 and 106 out of 117 individuals have an R^2 greater than 0.95.[20] In other words, subjects typically reported estimates that were very close to the best time perception power fit. Second, there is substantial heterogeneity across individuals in our sample. Indeed, 77 and 40 subjects had an estimated parameter $\hat{a}_i$ greater and smaller than 1, respectively. Similarly, 81 and 36 subjects had an estimated parameter $\hat{b}_i$ smaller and greater than 1, respectively. This suggests that crucial information is lost if we simply fit an aggregate perception function, and that constraining the function to be logarithmic (as in the majority of the psychophysics papers that follow the Weber–Fechner law) severely undermines the quality of the individual estimates.[21] Third, time perception parameters a_i and b_i are not independent across individuals. More precisely, we found a strong hyperbolic relation between the two parameters (see Fig. 2).[22] This means that both parameters cannot be studied

[18] However, it does not accommodate more general relationships, such as functions that are first convex and then concave.

[19] Following the standard definition, a value is considered an extreme outlier if it is at least 3 interquartile ranges below the first quartile or at least 3 interquartile ranges above the third quartile.

[20] R^2 is expected to be high since we fit two parameters with 9 observations. Still, since we impose $\theta_i(0) = 0$, our regression has 7 degrees of freedom. The subjects in Fig. 3 are representative of the fit. We also tried a linear model but the fit dropped substantially.

[21] Indeed, the literature has often specified the following time perception function: $\theta_i(\tau) = c_i \log(\tau + 1)$. This functional form performs substantially worse in our data: average adjusted R^2 is 0.81 (compared to 0.97), no individual (compared to 96) with an adjusted R^2 greater than 0.95, and 109 individuals (compared to 5) with an adjusted R^2 smaller than 0.90. A main problem with the logarithmic specification is its inability to capture convex time perceptions.

[22] The data is best fitted by the curve $a = -3.64 + \frac{4.7}{b}$ (p-value <0.001 for both parameters).

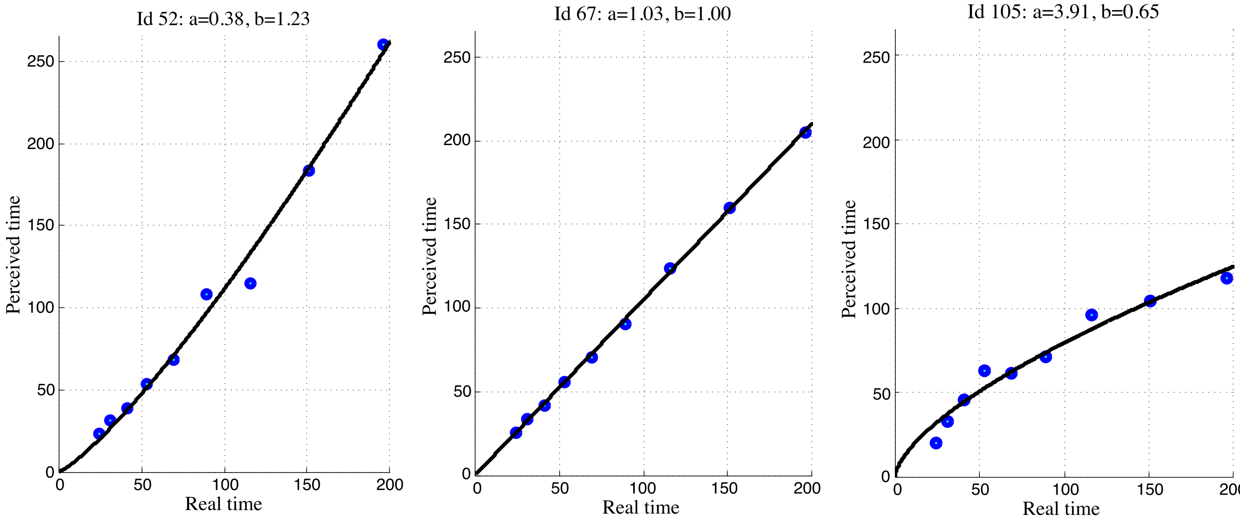

Fig. 3. Three examples of choices in the time perception task.

in isolation: an individual with a concave perception of time ($\hat{b}_i < 1$) is extremely likely to exhibit a steep reaction to time ($\hat{a}_i > 1$) and vice versa.

4. Time discounting ($\mathcal{TD}$)

We now present the theoretical framework and experimental results of our time discounting task. We refer to [AS] for extra details of the theory and estimation. Subject i chooses at time 0 to allocate a budget m between consumption at t, $c_{i,t}$, and consumption at $t+k$, $c_{i,t+k}$, continuously along a convex budget set. Denoting $(1+r)$ the gross interest rate, the budget constraint can be written as:

$$(1+r)c_{i,t} + c_{i,t+k} = m \tag{3}$$

We assume a time separable time discounting function $\Phi_i(t)$ of time t from the perspective of time 0, and a CRRA utility of money:

$$U_0(c_{i,t}, c_{i,t+k}) = \Phi_i(t)\frac{1}{\alpha_i}(c_{i,t})^{\alpha_i} + \Phi_i(t+k)\frac{1}{\alpha_i}(c_{i,t+k})^{\alpha_i} \tag{4}$$

where $\alpha_i > 0$ is the curvature parameter. To estimate the inter-temporal utility function of each individual, we arbitrarily restrict attention to quasi-hyperbolic discount functions, that is, functions of the form:

$$\Phi_i(t) = \begin{cases} \beta_i \delta_i^t & t > 0 \\ 1 & t = 0 \end{cases}$$

where $\delta_i \in (0,1)$ is the one period discount and $\beta_i > 0$ the time inconsistency parameter. Note that $\beta_i = 1$ corresponds to the standard exponential discounting model. A subject is time inconsistent when $\beta_i \neq 1$, exhibiting a present-bias when $\beta_i < 1$ and a future-bias when $\beta_i > 1$. The subject chooses $c_{i,t}$ and $c_{i,t+k}$ by maximizing (4) subject to (3). The optimal amounts are:

$$c^*_{i,0} = \frac{m}{(1+r) + \left((1+r)\beta_i\delta_i^k\right)^{\frac{1}{1-\alpha_i}}} \quad \text{and} \quad c^*_{i,t} = \frac{m}{(1+r) + \left((1+r)\delta_i^k\right)^{\frac{1}{1-\alpha_i}}} \tag{5}$$

We fitted the model to the data obtained from the time discounting task ($\mathcal{TD}$). For each individual, we estimated by NLS and MLE the parameters α_i, δ_i and β_i of the following regressions:

$$c_{i,0} = \frac{m}{(1+r) + \left((1+r)\beta_i\delta_i^k\right)^{\frac{1}{1-\alpha_i}}} + \varepsilon_{i,0} \quad \text{and} \quad c_{i,t} = \frac{m}{(1+r) + \left((1+r)\delta_i^k\right)^{\frac{1}{1-\alpha_i}}} + \varepsilon_{i,t} \tag{6}$$

where $\varepsilon_{i,0} \sim N(0, \sigma_i^2)$ and $\varepsilon_{i,t} \sim N(0, \sigma_i^2)$. Notice that variations in delay lengths k and interest rates $(1+r)$ allow for the identification of α_i and δ_i. Variations in starting times t allow for the identification of β_i.

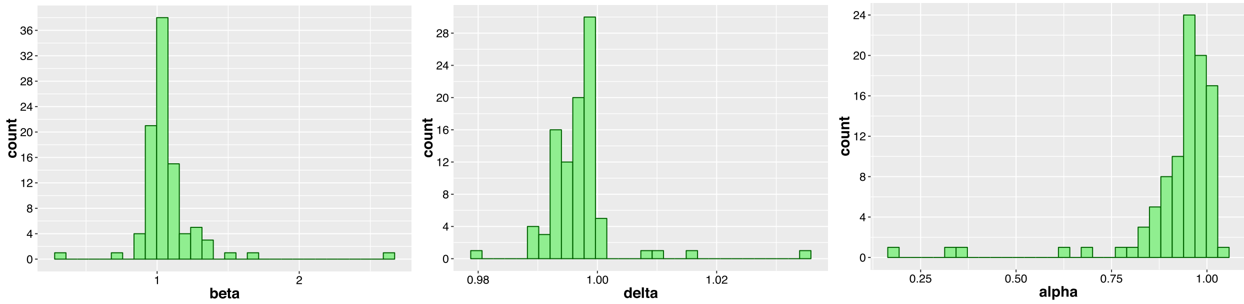

Fig. 4. Distribution of parameters in the time discounting task $(\hat{\beta}_i, \hat{\delta}_i, \hat{\alpha}_i)$.

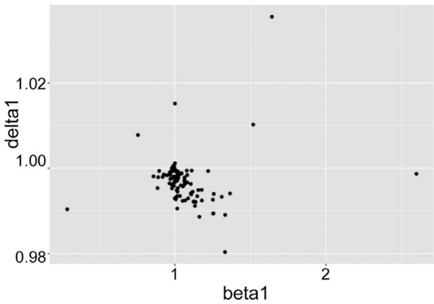

Fig. 5. Individual time discounting parameter estimates $(\hat{\beta}_i, \hat{\delta}_i)$.

From the 120 initial subjects, we removed 19 subjects who put all the tokens in the later option 44 or 45 times out of 45 (more than 97% of the time). Using a boxplot analysis and the same definition as in footnote 19, we identified six extreme outliers ($\hat{\alpha}_i \simeq 0$, $\hat{\alpha}_i > 2$ and/or $\hat{\beta}_i > 4$). These subjects were also excluded from the analysis.[23] Fig. 4 presents the distributions of the $(\hat{\beta}_i, \hat{\delta}_i, \hat{\alpha}_i)$ estimated parameters for the 95 remaining subjects using MLE while Fig. 5 graphically depicts the individual combinations of $\hat{\beta}_i$ and $\hat{\delta}_i$.

We obtained reasonable estimates. The estimates are also similar (and generally consistent) with those in [AS] (see their Fig. 3 in p. 3351).[24] In particular, the vast majority of our $\hat{\beta}_i$ estimates are close to 1, implying no evidence of present-biased behavior (if anything, just like in [AS] we observe a future bias, although this is likely due to small measurement errors). This result is not excessively surprising given the [AS] methodology employed, where "today" payments are out-lab and delayed by at least 2.5 hours. As expected, the overwhelming majority of the $\hat{\delta}_i$ estimates are between 0.99 and 1.0 and most of the $\hat{\alpha}_i$ estimates are above 0.85 (but still below 1).[25]

5. The relationship between time perception and time discounting

We have established that the time perception and time discounting of each individual i are well summarized by (a_i, b_i) and $(\beta_i, \delta_i, \alpha_i)$, respectively. In this section, we address the main question of the paper: the relationship between time discounting and time perception.

5.1. Main hypothesis

We hypothesize that objective delays between consumption dates are evaluated in a subjective manner, and the subjective estimates are used to choose between consumption options over perceived delays. We should therefore observe a relationship between the tendency to over- or under-represent true time and the willingness to delay gratification.

Formally, we posit that the discounting of an objective time interval for individual i, $\Phi_i(\cdot)$, corresponds to the time-weighting $f(\cdot)$ of the perceived length of that interval, $\theta_i(\cdot)$. Assuming for simplicity that $f(\cdot)$ is identical for all individuals, we have:

$$\Phi_i(t) = f(\theta_i(zt)) \quad \forall i \tag{7}$$

[23] Having non-reliable estimates for some subjects is not unusual in this type of exercise. For example, among the 97 subjects in [AS], 2 did not converge, 2 made automatic choices, 22 chose the later option more than 95% of the time and 7 were extreme outliers according to our definition.

[24] As expected, MLE and NLS give extremely similar estimates. The fit of the model is good. The average R^2 in our NLS estimation is 0.80.

[25] Aggregate estimates are also similar to [AS]: $\alpha = 0.922$, $\delta = 0.9969$ and $\beta = 1.068$ in our sample compared to $\alpha = 0.897$, $\delta = 0.9991$ and $\beta = 1.007$ in [AS].

where z is the conversion rate between units of time in the discounting and perception tasks.[26] According to (7), differences in discount functions across individuals are related to differences in their subjective perception of time. We impose the natural assumption that $f' < 0$: other things being equal, a reward is valued less if the perceived interval of time before it occurs is longer (in other words, for a given individual, a prize in a perceived short time is strictly better than a prize in a perceived long time). For now, however, we do not impose any specific functional shape for the decreasing function $f(\cdot)$.[27] This alone immediately implies:

$$\theta_i(zt) \gtreqless \theta_j(zt) \;\Leftrightarrow\; \Phi_i(t) \lesseqgtr \Phi_j(t) \tag{8}$$

The relationship in (8) formalizes in the simplest possible terms the intuitive idea that if an objective length of time zt is subjectively perceived as a longer interval by subject i than by subject j ($\theta_i(zt) > \theta_j(zt)$), then subject i is less willing than subject j to postpone a reward by that amount of time ($\Phi_i(t) < \Phi_j(t)$). The relationship, however, is not causal. In particular, it is consistent with the alternative interpretation that if subject k is more impatient than subject l, then subject k is also more likely to perceive a unit of time as subjectively longer than subject l.[28]

Hypothesis. *Subjects for whom one unit of time is perceived as longer are less willing to delay gratification by that amount of time.*

5.2. Main result

Recall that our time perception task elicits perceived durations in the range of minutes while our time discounting task elicits inter-temporal choices in the range of weeks. Ideally, we would like to determine whether the tendency to under or over-evaluate time in the estimated range of the $\mathcal{TP}$ task is related to the patience level in the estimated range of the $\mathcal{TD}$ task.

As mentioned in section 2.2, the non-linearity of the time perception function is potentially challenging: a subject with $\hat{b}_i < 1$ may overestimate short intervals and under-estimate long intervals whereas a subject with $\hat{b}_j > 1$ may exhibit the opposite pattern. Fortunately, given a power functional form $\theta(\cdot)$ and the empirical relationship between the estimates $\hat{a}_i$ and $\hat{b}_i$, it is possible to determine a time interval after which our subjects can be stably ranked in terms of their perception of time.[29]

In order to find such time interval in our data, we performed the following exercise. For each subject i and given his estimated parameters $(\hat{a}_i, \hat{b}_i)$, we evaluated his perception of an interval of length τ_x (that is, $\hat{\theta}_i(\tau_x) = \hat{a}_i \tau_x^{\hat{b}_i}$) and then ordered all subjects from $\max_i \{\hat{\theta}_i(\tau_x)\}$ to $\min_i \{\hat{\theta}_i(\tau_x)\}$. We repeated the same exercise for an interval of length $\tau_{x'}$. We then asked by how much this ranking changed between τ_x and $\tau_{x'}$. We found that the ranking of 28% of our subjects changed by more than 5 positions between 588 seconds (3 times the highest estimated interval) and 1 hour. This percentage dropped to 20% between 1 hour and 1 day, to 1% between 1 day and 7 days and to 0% thereafter. Overall, subjects in our sample can be ranked with stability regarding their perception of time for intervals above 1 hour.

We used a similar methodology to determine for which time intervals we can rank subjects by their level of impatience. We took the estimated $(\hat{\beta}_i, \hat{\delta}_i)$ parameters to determine for each individual i the value at date 0 of one unit of consumption at different dates t_x: 1 hour, 6 hours, 12 hours, 18 hours, 1 day and 7 days. We then ranked our subjects by their level of impatience at each of these dates. We found that 27% of the subjects changed ranks by more than 5 positions between 1 day and 7 days and none changed ranks between any of the shorter intervals. Subjects can then be ranked reasonably steadily in terms of their time discounting for all time horizons, and extremely steadily for horizons below 1 day. The result should not be surprising for the reader, given that in our sample the estimated $\hat{\beta}_i$ parameter of most subjects is close to 1 (indeed, if the discount rate of all subjects was exponential, no rank would ever change).

The overall conclusion of this exercise is that subjects in our sample can be ranked reasonably consistently in terms of time perception *and* time discounting for time intervals above 1 hour. Notice that there is also the issue of extrapolation: extreme downward extrapolation of time discounting intervals or upward extrapolation of time perception intervals result in excessively noisy, hence unreliable, measurements.

Given the exclusion criteria considered earlier for our estimations, we kept for the analysis the 92 subjects for whom we obtained accurate and reasonable estimates in both the $\mathcal{TP}$ and $\mathcal{TD}$ tasks.[30] We then considered the smallest upward extrapolation for which subjects' time perception could be ranked steadily and, at the same time, the largest downward extrapolation that provided meaningful estimates for time discounting, namely 1 hour (1 h). Similarly, we considered the

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[26] In our case, given we formulated the time perception task in seconds ($\tau = 1$ second) and the time discounting task in days ($t = 1$ day), we have $z = 60 \times 60 \times 24 = 86{,}400$.

[27] In section 7 we propose a quasi-hyperbolic specification for the time-weighting function and provide a structural estimation.

[28] There are other interesting issues not captured by this simple formulation, such as the role of anticipation and regret on subjective time evaluation.

[29] Extrapolating time perception presupposes that the parameter estimates remain accurate for predicting time perception out of the range of our measurements. As noted earlier, this assumption is plausible given the empirical knowledge and theoretical framework developed in psychophysics.

[30] Recall that we excluded 3 extreme outliers in $\mathcal{TP}$, 6 extreme outliers in $\mathcal{TD}$ and 19 subjects with insufficient variance in $\mathcal{TD}$. Removing 23% of the sample is not ideal. In section 6, we discuss the robustness of our results to other (more or less stringent) sample specifications.

Table 1
Correlation between time perception estimates ($\hat{a}_i(zt)^{\hat{b}_i}$) and time discounting estimates ($\hat{\beta}_i\hat{\delta}_i^t$).

Measure	PCC	p-value
1 d ($\mathcal{TP}$) – 1 d ($\mathcal{TD}$)	−0.22	0.038
1 h ($\mathcal{TP}$) – 1 d ($\mathcal{TD}$)	−0.26	0.013
1 h ($\mathcal{TP}$) – 1 h ($\mathcal{TD}$)	−0.26	0.013
1 d ($\mathcal{TP}$) – 1 h ($\mathcal{TD}$)	−0.22	0.036

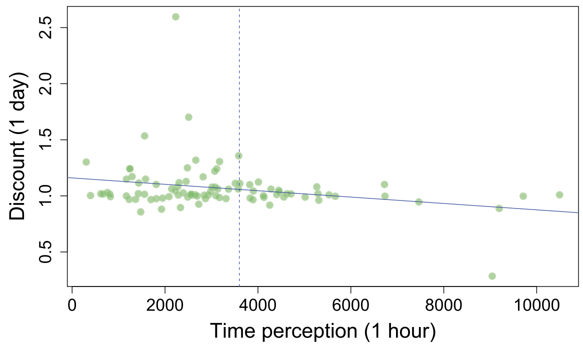

Fig. 6. Estimates of time perception at 1 hour and time discounting at 1 day.

smallest downward extrapolation for which subjects' time discounting could be ranked steadily and, at the same time, the largest upward extrapolation that provided meaningful estimates for time perception, namely 1 day (1 d). We then determined the correlation between discounting and perception at the individual level over that range. Outcomes are summarized in Table 1 and the Result.

Result. There is a negative and statistically significant correlation between time perception and time discounting in the 1 hour to 1 day range.

The result provides support for our Hypothesis.[31] Impatience is associated with the overestimation of perceived time, as predicted by our simple model. Subjects who produce a higher interval in the time perception task tend to consume earlier.[32] Fig. 6 represents the scatterplot of predicted time perception at 1 hour and predicted time discounting of 1 day.[33]

Overall, the data provides support for the link between subjective perception of time and impatience, and the existence of a time-weighting function $f(\cdot)$ that maps perceived time into discount rates. From Fig. 6, it seems that the negative relationship is convex and that there is substantial heterogeneity across individuals in the valuation of future rewards and the subjective evaluation of delays.[34]

It is important to emphasize that the estimates $(\hat{a}_i, \hat{b}_i)$ on one hand and $(\hat{\beta}_i, \hat{\delta}_i)$ on the other are obtained from independent datasets, $\mathcal{TP}$ and $\mathcal{TD}$. There is a priori no reason (other than the endogenous relationship emphasized by our model) why the measures constructed from these two datasets should correlate. And yet, there is evidence that the mechanisms underlying time perception and time discounting are linked. Furthermore, the data collected in each task is noisy, which weakens the correlation between perception and discounting. This means that the correlation obtained is likely to be underestimated.[35]

To better investigate the relationship between time perception and time discounting, we run a regression between time discounting of 1 day and time perception of 1 hour. There is an obvious difficulty, since the exercise requires choosing which of the two measures is the dependent and which one is the explanatory variable. We include several measures of

[31] Results are robust to other horizons provided we do not excessively increase the extrapolation. For example, PCC $= -0.25$, p-value $= 0.019$ for the correlation between 1 h ($\mathcal{TP}$) and 7 d ($\mathcal{TD}$) and PCC $= -0.21$, p-value $= 0.040$ for the correlation between 1 h ($\mathcal{TP}$) and 14 d ($\mathcal{TD}$).

[32] We perform the same analysis using a Bayesian method and find similar results: bayesian PCC $= -0.08$ (respectively -0.12, -0.13 and -0.08) with a likelihood that the correlation is negative of 0.75 (respectively 0.85, 0.87 and 0.76) for the correlations in the order displayed in Table 1.

[33] The reader may wonder whether the result is driven by a few observations that are "visually distant" from others in the graph. If (for ad hoc reasons) we decide for example to exclude from our sample the 3 subjects who made the most extreme choices, we obtain the same correlation magnitude and significance. More rigorous robustness checks are presented in section 6.

[34] Notice also that the discounting is above 1 for many subjects, which is obviously unreasonable. The reason is that many $\hat{\beta}_i$-estimates are above 1, so the downward extrapolation of the parameters to one day results in unrealistically high patience levels. Remember, however, that the main objective of our analysis is not to estimate levels of time discounting and time perception but to be able to perform comparisons across individuals. Under no extrapolation (e.g., a discounting of 5 weeks), then more than 90% of subjects exhibit reasonable levels of preference for the present.

[35] Experimental data is intrinsically noisy and subject to measurement errors. Given this noise, the true coefficient of correlation between time perception and time discounting is by construction higher than the coefficient of correlation between their noisy measurements (see Gillen et al., 2015 for a statistical method that corrects for measurement errors).

Table 2
Regression analysis of 1-day discount rate ($\mathcal{TD}$).

	Model 1	Model 2
1 h ($\mathcal{TP}$)	−2.844e−05*	−3.735e−05**
Filler score	–	3.267e−03
GPA score	–	−5.677e−02
IQ score	–	2.276e−02
Male	–	−1.739e−02
Non English	–	1.278e−02
Preference	–	−3.147e−01
Retrospective	–	7.963e−02
constant	1.160***	1.30***
Adj. R^2	0.06	0.09

Significance levels: * 5%, ** 1%, *** 0.1%.

Table 3
Spearman/Kendall rank correlation between time perception and time discounting (one-sided tests).

Measure	Spearman	p-value	Kendall	p-value
1 d ($\mathcal{TP}$) – 1 d ($\mathcal{TD}$)	−0.18	0.045	−0.12	0.039
1 h ($\mathcal{TP}$) – 1 d ($\mathcal{TD}$)	−0.17	0.046	−0.13	0.038
1 h ($\mathcal{TP}$) – 1 h ($\mathcal{TD}$)	−0.17	0.057	−0.11	0.051
1 d ($\mathcal{TP}$) – 1 h ($\mathcal{TD}$)	−0.16	0.057	−0.12	0.048

relevance. First, subjects who overestimate time should in principle complete more filler tasks. A simple correlation exercise indicates that subjects who report longer durations complete more filler tasks (PCC = 0.54, p-value <0.001). They are successful more often (PCC = 0.45, p-value <0.001) but also make more mistakes (PCC = 0.21, p-value <0.036). Overall, their performance is not significantly different. We use the number of filler tasks completed successfully (*Filler score*) as a control. Second, it may also be the case that cognitive abilities, as measured by IQ and GPA scores affect discounting attitudes. We therefore include the GPA score reported by subjects (*GPA score*) and the results of our cognitive ability test (*IQ score*). We also include a dummy for gender (*Male*), a dummy for first language (*Non English*), the percentage bias in the retrospective task (*Retrospective*) as well as $\hat{\alpha}_i$ estimates to control for intrinsic preference attitudes (*Preference*). Table 2 reports our results. Model 1 corresponds to the simple correlation analysis (and yields the same significance). Model 2 includes the various control measures.[36]

Even though the linear model explains only very partially the variance in the data, it allows us to identify significant and insignificant correlations between our measures. The results indicate that the correlation between perception and discounting emphasized previously persists in the regressions, with an increase in significance when we include control variables.[37] By contrast, there is no evidence that any of the other measures has an impact on time discounting in our sample.[38]

6. Robustness

We performed a number of robustness checks. The conclusion is that our main findings are, to a large extent, robust to different specifications. The reader who is not interested in the details can skip the section without loss.

Rank Correlations. We have argued that correlations and statistical significance are similar in the 1 hour to 1 day range because they are driven mainly by rankings (not point estimates), and we have shown that rankings are stable in this interval period. However, it can be useful to ignore values and simply determine rank correlations between time perception and time discounting. Table 3 presents the Spearman and Kendall rank correlation coefficients. We test for the hypothesis that correlations are negative and report one-sided tests.

While the general results are robust, both the correlation coefficients and the statistical significance are weaker when we consider the correlation of ranks instead of values.

Time perception sensitivity. A possible issue regarding time perception estimates is that they might be excessively sensitive to the (few) observations obtained in the time perception task. If this is the case, the extrapolation to unit of times outside the range of our observations may be problematic. To investigate this issue, we removed the longest interval (196 s) and

[36] The effects are preserved if we replace the explanatory variable 1 h ($\mathcal{TP}$) by 1 d ($\mathcal{TP}$) or if we use as the dependent variable 1-hour ($\mathcal{TD}$) instead of 1-day ($\mathcal{TD}$). Regressions results are very similar, except that significance is increased when shorter time intervals are used.

[37] We performed a number of robustness checks. First, to account for outliers, we ran a robust regression using M estimation (Huber method) and we found that the coefficient for $\mathcal{TP}$ was significantly negative (p-value = 0.017 for Model 1 and p-value = 0.004 for Model 2). Second, we ran a Bayesian regression and came to the same conclusion (p-value = 0.013 for Model 1 and p-value = 0.003 for Model 2).

[38] We did not find an effect of gender, IQ or GPA on time perception either. The absence of gender differences departs from the results in recent studies on time discounting (Dittrich and Leipold, 2014) and time perception (Koglbauer, 2015).

Table 4
Correlation between time perception estimates derived from the multiplicative noise model and time discounting estimates.

Measure	PCC	p-value
1 d ($\mathcal{TP}$) – 1 d ($\mathcal{TD}$)	−0.21	0.041
1 h ($\mathcal{TP}$) – 1 d ($\mathcal{TD}$)	−0.25	0.015
1 h ($\mathcal{TP}$) – 1 h ($\mathcal{TD}$)	−0.25	0.014
1 d ($\mathcal{TP}$) – 1 h ($\mathcal{TD}$)	−0.22	0.039

estimated the parameters $(\hat{a}_i^8, \hat{b}_i^8)$ with the 8 remaining observations. We found that, on average, the curvature of the time perception function was under-estimated (0.84 vs. 0.93) indicating that the last observation carried significant information.[39] When we predicted the response of each participant to the missing 196 s interval, we found that the predictions co-varied with the actual responses, although less than perfectly (PCC = 0.75, p-value <0.001). With these new estimates, subjects in our sample could be ranked with stability on their perception of time for intervals above 20 minutes. We therefore extrapolated time perception to 20 minutes and correlated 20 m ($\mathcal{TP}$) and 1 d ($\mathcal{TD}$). Despite the loss in precision, the correlation was still negative. The significance of point correlation was reduced, but not that of rank correlations (PCC = −0.15, p-value = 0.077; Spearman = −0.19, p-value = 0.033; Kendall = −0.13, p-value = 0.034; one sided-tests). For comparison, we also extrapolated time perception to our 1 hour benchmark and we obtained similar results (PCC = −0.12, p-value = 0.13; Spearman = −0.19, p-value = 0.033; Kendall = −0.13, p-value = 0.030; one sided-tests). We also conducted the same analysis by removing the shortest interval, then the median interval. As expected, these observations carried less information than the longest interval. Indeed, the correlation between 1 h ($\mathcal{TP}$) and 1 d ($\mathcal{TD}$) was similar to the one obtained in Table 1 (PCC = −0.28, p-value = 0.004; Spearman = −0.22, p-value = 0.016; Kendall = −0.15, p-value = 0.015 when the shortest was removed; PCC = −0.24, p-value = 0.01; Spearman = −0.20, p-value = 0.025; Kendall = −0.15, p-value = 0.020 when the median was removed; one sided-tests). The overall conclusion of this robustness exercise is that, while each interval (and especially the longest one) is important in determining the shape of the time perception function, the negative correlation between time perception and time discounting does not hinge upon one single measure.

Alternative noise structure. To assess whether the additive noise structure assumed in the paper has a crucial impact on our time perception estimates, we ran a new set of regressions using a different (multiplicative) noise formulation:

$$y_{is} = a_i \tau_s^{b_i} u_{is} \tag{9}$$

Linearizing, we estimated parameters from the equation:

$$\log(y_{is}) = \log(a_i) + b_i \log(\tau_s) + \log(u_{is}) \tag{10}$$

where $\log(u_{is}) \sim N(0, \sigma_i^2)$. This new model fitted the data significantly worse. The R^2 was greater with the new model for only 4 out of 120 participants. However, the parameters estimated with the additive and multiplicative error structures were strongly correlated (PCC between $\hat{a}_i$ estimates = 0.70, p-value <0.001; PCC between $\hat{b}_i$ estimates = 0.72, p-value <0.001). After removing outliers in a similar fashion as before, we obtained a set of 90 subjects with reliable estimates in both tasks. We replicated the same analysis as in Table 1 and we report the results in Table 4.

As we can from Tables 1 and 4, correlations and statistical significance are remarkably similar in both cases, with the additive model slightly outperforming the multiplicative one.

Analysis with and without outliers. To check whether our results are sensitive to the exclusion criteria used to retain our 92 subjects, we construct two new samples. First, a large sample, where we reintroduce subjects who exhibit little variance in behavior in the discounting task (but not the 2 subjects who exhibit no variance at all). Given some of them also have extreme estimates, the large sample is composed of 106 subjects (88% of participants). Second, a small sample that excludes outliers which are less extreme compared to the main sample.[40] The small sample is composed of 78 subjects (65% of participants). It corresponds to the dataset analyzed in detail in the previous version of the paper. Correlations are reported in Table 5 and the results of the regression analysis are collected in Table 6.

Time discounting and time perception are correlated in the 1 hour to 1 day range both when we consider the small and the large samples.[41] Overall, the outliers do not seem to amplify or diminish the main finding obtained earlier. The results are also supported by a robust regression approach (p-value = 0.007 for the small sample and p-value = 0.051 for the large

[39] More precisely, denoting $\theta^8(\cdot)$ the perception of time (in seconds) resulting from our new estimates, we found that $\theta_i^8(1\text{ h}) = 853 + 0.7 \times \theta_i(1\text{ h})$ (p-value <0.001).

[40] The exclusion criteria were $a_i < 0.16$, $a_i > 8$, $\alpha_i \simeq 0$, $\alpha_i > 1$, $\beta_i < 0.3$, $\beta_i > 4$, $\delta_i > 1$ and subjects who put all the tokens in the later option 44 or 45 times out of 45. We also omitted subjects for whom we had convergence problems in the estimation (we had to change the criteria of convergence to find reliable estimates).

[41] We also considered other subsets. If, instead of reintroducing subjects exhibiting little variance in behavior, we reintroduce subjects whose time perception estimates are extreme outliers (new sample size is 95 subjects), then PCC = −0.23 (p-value = 0.028). If we reintroduce subjects whose time discounting estimates are extreme outliers (new sample size is 97 subjects), then PCC = −0.20 (p-value = 0.054).

Table 5
Correlation between time perception estimates and time discounting estimates varying the exclusion criteria.

Measure	Small sample (78)		Large sample (106)	
	PCC	p-value	PCC	p-value
1 d ($\mathcal{TP}$) – 1 d ($\mathcal{TD}$)	−0.25	0.027	−0.20	0.038
1 h ($\mathcal{TP}$) – 1 d ($\mathcal{TD}$)	−0.23	0.043	−0.24	0.013
1 h ($\mathcal{TP}$) – 1 h ($\mathcal{TD}$)	−0.23	0.043	−0.24	0.013
1 d ($\mathcal{TP}$) – 1 h ($\mathcal{TD}$)	−0.25	0.026	−0.20	0.036

Table 6
Regression analysis of 1-day discount rate ($\mathcal{TD}$) varying the exclusion criteria.

	Small sample	Large sample
1 h ($\mathcal{TP}$)	−2.145e−05**	−3.541e−05**
Filler score	1.429e−03	2.561e−03
GPA score	5.198e−02	−1.143e−02
IQ score	2.235e−03	2.345e−02
Male	2.184e−02	−3.742e−02
Non English	1.148e−02	−2.358e−02
Preference	2.119e−02	−1.305e−01
Retrospective	5.881e−02	4.053e−02
constant	0.838***	1.044***
# obs.	78	106
Adjusted R^2	0.091	0.039

Significance levels: * 5%, ** 1%, *** 0.1%.

Table 7
Correlation between time perception estimates and time discounting estimates (Weibull or Best fit).

Measure	Weibull		Best fit	
	PCC	p-value	PCC	p-value
1 d ($\mathcal{TP}$) – 1 d ($\mathcal{TD}$)	−0.27	0.009	−0.23	0.026
1 h ($\mathcal{TP}$) – 1 d ($\mathcal{TD}$)	−0.34	0.001	−0.26	0.011
1 h ($\mathcal{TP}$) – 1 h ($\mathcal{TD}$)	−0.41	0.001	−0.23	0.026
1 d ($\mathcal{TP}$) – 1 h ($\mathcal{TD}$)	−0.34	0.001	−0.21	0.044

sample) and a bayesian approach (p-value = 0.010 for the small sample and p-value = 0.037 for the large sample). Again, all results are preserved if we explain 1 hour ($\mathcal{TD}$) instead of 1 day ($\mathcal{TD}$).

Weibull specification. Since the quasi-hyperbolic discount function does not always capture accurately the discounting pattern of individuals, we follow Andersen et al. (2014) and consider the parametric specification proposed by Read (2001) to account for time subadditivity, namely the Weibull function:

$$\Phi_i^W(t) = \exp(-\mu_i t^{(1/s_i)})$$

with $\mu_i > 0$ and $s_i > 0$. This is comparable to quasi-hyperbolic discounting in that it is also a two-parameter function that boils down to time-consistent exponential discounting when one of the parameters takes a specific value (in our formulation, when $s_i = 1$). For each individual, we fitted the parameters of the new discounted utility function and we obtained estimates for α_i, μ_i and s_i. We found that the Weibull and quasi-hyperbolic models perform on aggregate very similarly according to the Akaike Information Criterion (AIC).[42] We then conducted the same correlation analysis as we did before with the 92 subjects of the main model under two scenarios. First, assigning all the subjects to the Weibull discounting function (Weibull). Second, assigning each subject to the model that fitted best, quasi-hyperbolic or Weibull (Best fit). Table 7 reports the same information as Table 1 for these two cases.

While the fit of the two models is similar, the correlation coefficient is higher and the statistical significance stronger with the Weibull specification than with the quasi-hyperbolic or the best fit. This, however, should not be overemphasized as

We can also exclude subjects who are close to the 80% threshold performance in the filler task, as they may have artificially delayed finishing the time perception task so as to increase the number of filler tasks. For exclusion criteria 77% to 83% and 79% to 81%, we get PCC = −0.25 (p-values = 0.020 and 0.019 respectively).

Finally, we can also exclude subjects with poor performance. If they anticipate that they are unlikely to be paid, they have reduced incentives to measure time accurately. For exclusion criteria <80% and <75%, we get PCC = −0.27 (p-values = 0.013 and 0.010 respectively).

[42] To be precise, the Weibull model performed slightly better. However, this was due to two subjects who were significantly better captured by that model. Without considering these two subjects, the average performance of the two models was almost indistinguishable (the mean AIC was 237 for the quasi-hyperbolic model and 234 for the Weibull model).

Table 8
Correlation between time perception estimates ($\hat{a}_i(zt)^{\hat{b}_i}$) and discounted utility estimates ($\hat{\beta}_i\hat{\delta}_i^t \frac{1}{\hat{\alpha}_i}(c)^{\hat{\alpha}_i}$).

Measure	PCC	p-value
1 d ($\mathcal{TP}$) – 1 d ($\mathcal{TD}$)	−0.23	0.026
1 h ($\mathcal{TP}$) – 1 d ($\mathcal{TD}$)	−0.28	0.008
1 h ($\mathcal{TP}$) – 1 h ($\mathcal{TD}$)	−0.28	0.007
1 d ($\mathcal{TP}$) – 1 h ($\mathcal{TD}$)	−0.23	0.025

it is driven by a small subset of individuals. Overall, the negative correlation between time discounting and time perception seems robust to the specification of the discounting function.

Time perception and discounted utility. Our structural approach implicitly assumes that the utility model is correct. If, contrary to our specification, utility and discount are not fully separable in the "true model" that generates the data, $\hat{\alpha}_i$ estimates may capture some elements of discount while $\hat{\beta}_i$ and $\hat{\delta}_i$ estimates may capture some elements of utility. To address this possibility, we correlate time perception with the discounted utility of consumption of several units. Formally, we take the CRRA utility representation of our model and extend the relationship in equation (8) to:

$$\theta_i(zt) \gtrless \theta_j(zt) \;\Leftrightarrow\; \beta_i\delta_i^t \frac{1}{\alpha_i}(c)^{\alpha_i} \lessgtr \beta_j\delta_j^t \frac{1}{\alpha_j}(c)^{\alpha_j} \tag{11}$$

In words, the hypothesis is that if a subject perceives one unit of time as a longer interval than another subject, he will values less the *consumption of c units* after that amount of time. Table 8 reports the same correlation exercise as in Tables 1 and 7 for the valuation of 5 units of consumption.[43]

The statistical significance of the correlations is stronger compared to Table 1, suggesting that the perception of time affects the evaluation of delayed rewards rather than simply the evaluation of delays.

Non-parametric test. Our data does not allow us to make meaningful non-parametric tests based on general patterns of consumption (such as the percentage of tokens allocated to early consumption) and general tendency to underestimate time (such as the proportion of times the subject underestimated the true time). Here is why. The non-linearity of the perception function strongly indicates that the estimated curvature is more important to predict over- vs. under-estimation of long intervals than the observations in the computed range. Also, the decision to allocate tokens early depends on both the degree of patience and the concavity of the utility function, so the percentage allocated to the earliest date is not capturing well the attitude of a person towards delays. However, one simple analysis we can conduct is to divide our sample between those who underestimate 1 hour and those who overestimate 1 hour. We find that the average discount of 1 day is different for these two groups (1.11 vs. 0.99, p-value of a two-sample t-test = 0.005, difference of means greater than 0 with probability 0.934 with a Bayesian t-test).[44] In words and reinforcing previous findings, subjects who underestimate 1 hour exhibit on average more patience than those who overestimate it.

Correlations between actual measurements. Our result relies on the assumption that time perception estimates obtained through measurements in the range of seconds to minutes can be extrapolated to one hour and above. Even though it is a plausible assumption, one would like to demonstrate a relationship between time perception and time discounting based on actual experimental measurements. Remember, however, that the main reason to consider intervals of time of at least one hour is to ensure that the rank of subjects in terms of their time perception is stable.

We can follow a different strategy. Instead of determining the interval after which the rank of the majority of subjects is stable, we can restrict attention to the subset of subjects who can be stably ranked after one of the measured intervals. As the time delay decreases, the corresponding subset shrinks. We therefore select the largest interval for which reports are made, namely 196 s, and retain for the analysis the 58 subjects whose rank is stable after that interval. Columns 1 and 2 in Table 9 presents the Pearson's correlation for these individuals between the estimated time perception in the longest interval effectively measured (196 s) and the discounting at 1 day or 1 hour (rows 1 and 2). It also presents the same correlation using the actual reports at 196 s rather than the estimations (rows 3 and 4). Columns 3 and 4 in Table 9 report the Bayesian correlation and the posterior probability that the correlation is negative.

The result indicates that, *for subjects with stable ranking*, the negative correlation between their time perception on a measured interval and their time discounting extrapolated at 1 hour or 1 day is stronger and statistically more significant than the one documented in Table 1. In other words, as long as the underlying time perception of subjects is stable on the measured time interval – even if such interval is small – it is possible to predict their impatience from it. In Appendix A3, we present the same information as in Table 9, except that we use the large sample of 106 individuals. The PCC remains

[43] Naturally, units matter for this exercise. We used 5 units because it roughly corresponds to the dollar amount they had to evaluate in $\mathcal{TD}$. We tried other units around that number and found similar results.

[44] We obtained similar result when dividing the sample between those who underestimate one day and those who overestimate it (1.10 vs. 0.99, p-value of a two-sample t-test = 0.007, difference of means greater than 0 with probability 0.917 with a Bayesian t-test).

Table 9
Correlation between time perception estimates or time perception reports at 196 s and time discounting estimates for subjects with stable ranking.

Measure	Subjects with stable rank (58)			
	PCC	p-value	Bayes Corr.	prob. of (-)
196 s ($\mathcal{TP}$) – 1 d ($\mathcal{TD}$)	−0.41	0.002	−0.20	0.91
196 s ($\mathcal{TP}$) – 1 h ($\mathcal{TD}$)	−0.40	0.002	−0.20	0.91
196 s report – 1 d ($\mathcal{TD}$)	−0.39	0.002	−0.18	0.89
196 s report – 1 h ($\mathcal{TD}$)	−0.39	0.002	−0.18	0.89

negative but becomes statistically not significant. The probability that the Bayesian correlation is negative remains significant. The decreased significance is not surprising since adding these subjects amounts to introducing individuals whose ranking changes between the last estimated interval and the period of interest.

As a general conclusion, the analyses presented in section 6 suggest that the relationship between perceived time and preference for the present is robust: people who overestimate objective intervals of time are more inclined to consume early.

7. Individual time-weighting function

From the $\mathcal{TP}$ dataset, we showed that individual i's time perception is well summarized by $\theta_i(zt) = a_i(zt)^{b_i}$. From the $\mathcal{TD}$ dataset, we found that individual i's discount function can be approximated by $\Phi_i(t) = \beta_i \delta_i^t$. Our analysis revealed that the discount function can be interpreted as the time weighting of perceived time. Even though a given perceived time interval is reached for different true time intervals for different individuals, its weighting is similar across individuals. Overall, we have shown that the data can be summarized reasonably well by a *common* weighting function $f(\cdot)$, that transforms perceived times into discount rates $\Phi_i(t) = f(\theta_i(zt))$. Still, we noted some individual differences. The purpose of this section is to investigate this heterogeneity in more detail.

To do so, we posit that the weighting function that transforms perceived time into discount has the same structure for every participant but it is parametrized individually. More precisely, we assume:

$$\Phi_i(\theta_i(zt)) = {\beta'}_i {d'}_i^{a_i(zt)^{b_i}}$$

This quasi-hyperbolic formulation is the same as the one we used to estimate discounts, except that participants are now assumed to discount payoffs with respect to their perception of time rather than the true time. It also uses one second as the unit interval of time, so d'_i can be interpreted as the discount per second. Notice that if we set $\delta'_i \equiv {d'_i}^{z^{b_i}}$, then we can rewrite the previous function using one day as the (standard) unit of time:

$$\Phi_i(\theta_i(t)) = \beta'_i {\delta'_i}^{a_i(t)^{b_i}}$$

Our objective is to revisit the $\mathcal{TD}$ data and to propose a new discounting model driven by perceived time rather than true time. Following the very same optimization procedure as in section 4, the optimal consumption of individual i at date t is:

$$c^{**}_{i,0} = \frac{m}{(1+r) + \left((1+r)\beta'_i {\delta'_i}^{a_i(k)^{b_i}}\right)^{\frac{1}{1-\alpha'_i}}} \quad \text{and} \quad c^{**}_{i,t} = \frac{m}{(1+r) + \left((1+r){\delta'_i}^{a_i(t+k)^{b_i} - a_i(t)^{b_i}}\right)^{\frac{1}{1-\alpha'_i}}}$$

To make the estimation comparable to that in section 4, we do not estimate all 5 parameters again. Instead, we import the time perception parameters $\hat{a}_i$ and $\hat{b}_i$ estimated from the dataset $\mathcal{TP}$ and we estimate by MLE the remaining three parameters $(\hat{\beta}'_i, \hat{\delta}'_i, \hat{\alpha}'_i)$ in dataset $\mathcal{TD}$ exactly as we did before. Fig. 7 presents the distributions of the $(\hat{\beta}'_i, \hat{\delta}'_i, \hat{\alpha}'_i)$ estimated parameters.

A comparison between Fig. 4 and Fig. 7 suggests that the distribution of estimated parameters are remarkably similar when we consider perceived time rather than true time. We find that $\hat{\beta}_i$ and $\hat{\beta}'_i$ are positively correlated (PCC = 0.77, p-value <0.001) and so are $\hat{\alpha}_i$ and $\hat{\alpha}'_i$ (PCC = 0.91, p-value <0.001). Said differently, participants have very similar time inconsistency and curvature estimates in both models. Interestingly, even though the distributions of $\hat{\delta}_i$ and $\hat{\delta}'_i$ are very similar, the parameters are not significantly correlated (PCC = 0.13, p-value = 0.228). This is not surprising because $\hat{\delta}'_i$ is now applied to perceived time and individuals are highly heterogeneous in their time perception. The new parameter is therefore adjusting for the individual perception biases. Formally, the counterpart of $\hat{\delta}_i^t$ is now $\hat{\delta}_i'^{at^b}$ so, unlike for parameters β and α, correlations of δ across models cannot be studied independently of t (for that very same reason, $\hat{\delta}'_i$ cannot be interpreted as the daily discount factor). Overall, the model introduced here is a reinterpretation of the standard quasi-hyperbolic discounting model in terms of perceived time. According to AIC, both models perform very similarly (AIC = 236 for the initial model and AIC = 237 for the new model), suggesting that information regarding time perception is useful to describe their discounting attitude.

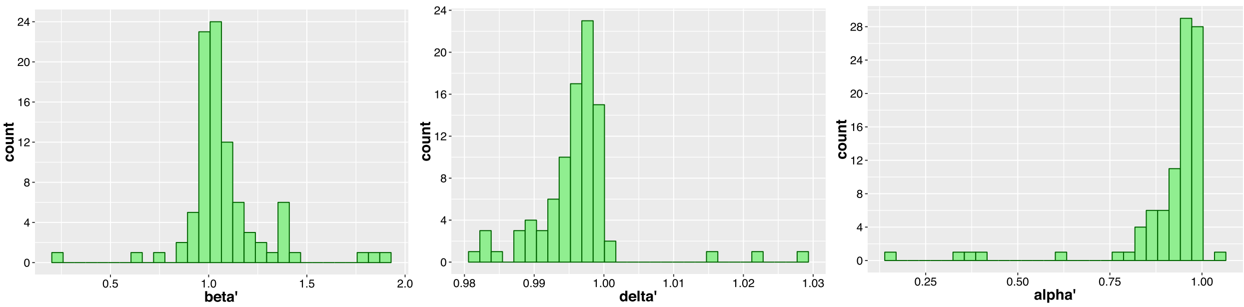

Fig. 7. Distribution of parameters $(\hat{\beta}_i', \hat{\delta}_i', \hat{\alpha}_i')$.

The main conclusion of this section is that a model in which time perception affects the way we perceive delays and evaluate future rewards is plausible. On the other hand, it also implies that the standard model based on objective delays is a reasonable approximation too. Either way, the heterogeneity we observe suggests that other mechanisms are likely to be at play, so we cannot predict with certainty how a subject will discount future rewards based on how he reports experienced time. In Appendix A4 we take another look at heterogeneity by clustering individuals according to their estimated time perception and time discounting at 1 day.

8. Concluding remarks

This paper provides experimental evidence on the relationship between time perception and time discounting. Our data reveals a negative correlation between the two: subjects who provide higher estimates of time are less willing to delay gratification. This result suggests that our ability to delay consumption is related to our mental representation of delays between now and the future. Our result is also consistent with the hypothesis that an underlying internal clock governs time representations irrespective of the unit of time.[45]

Time discounting and time preferences have been traditionally considered as primitives in decisions involving time trade-offs. Our evidence suggests that time keeping mechanisms account partially for those decisions. Therefore discount functions should be seen as reduced forms that capture time keeping rather than primitives. Even though more research is needed to evaluate the details of the relationship between time perception and inter-temporal trade-offs, our study offers a first promising look into the black box.

Our result is consistent with a growing body of the literature that studies the underlying mechanisms of time related evaluations. Prospective timing has been associated with working memory, a function performed by the dorsolateral pre-frontal cortex (dlPFC) (Grondin, 2010; Lewis and Miall, 2006). Time representation has been shown to involve the striatum and basal ganglia (Ivry and Spencer, 2004; Meck, 2005). Recent evidence in neuroscience supports the idea that the dlPFC and the striatum are also implicated in time discounting (van den Bos et al., 2014). This provides a rationale for why time perception and time discounting should be related, as indicated by our data.

The result is also in line with findings obtained in the time discounting and time perception literatures over the life cycle. It has been shown that the subjective perception of the passing time tends to speed up with age, so that people increasingly underestimate time as they age (Block et al., 1999; Coelho et al., 2004). In parallel, children succumb to immediate gratification while older adults are typically willing to wait for rewards (Green et al., 1999; Löckenhoff et al., 2011). In other words and consistent with our findings, children are impatient and overestimate time whereas older adults are patient and underestimate time. Interestingly, the dlPFC, which has been shown to be at the core of time related judgments, is late to develop in children (Casey et al., 2005) and early to age (Raz et al., 2005). These points taken together suggest that the relationship between time perception and time discounting and the changes over the life cycle are no coincidence.

The correlation between perceived time and discounting indicates that subjects judge future delays based on current experiences. While our result does not prove causality between time perception and time discounting, it suggests that manipulating the current perception of time may affect inter-temporal decisions. The literature in psychophysics has already shown that time perception can be altered by a long series of stressors including changes in body temperature and environmental factors (Droit-Volet and Meck, 2007; Meck and MacDonald, 2007). On the discounting side, Ebert and Prelec (2007) have demonstrated that time preferences can also be affected by pressure and attention manipulations. A natural alley for future research is to investigate wether the relationship between time perception and time discounting holds with manipulations and whether it is possible to induce patient choices in an efficient and ecologically valid way. Indeed, most of the policies that target behavior rely on information transmission (tell people that saving for the future is good) or exploit behavioral biases (changing the default option). An alternative is to design environments that promote beneficial

45 This conclusion is strengthened by the fact that, according to our retrospective task, there is also a relationship between retrospective and prospective time evaluation (see Appendix A2). Overall, we conjecture that a common mechanism is involved in all time related evaluations.

decision-making. If time perception is an important driver of time-related decisions and if it can be manipulated, creating environments in which better time evaluations are made should improve the way people make inter-temporal trade-offs.

Our study follows the methodology and definitions developed in psychophysics. In that literature, time perception refers to the relationship between the actual and the perceived change in physical time. Subjective time is measured using timing tasks and it refers to the interval people report when they target an objective time interval given by the experimenter. This has two implications. First, our measure of time perception does not refer to a loose statement about how long a delay "feels" to a subject. For that approach, one could develop different definitions as well as different measures of subjective time (e.g., verbal assessments). However, this would depart from the methodologies in both psychophysics and economics. Second, there is a difference between uncorrected time perception, referring to the intrinsic evaluation of a time interval, and corrected time perception, referring to the correction a subject might apply to make a report as accurate as possible despite his intrinsic evaluation. In our study, we only analyze time perception post-correction. It shall be noted that the same distinction may be relevant for time discounting and, there again, our study applies to the post-correction case.

Finally, the idea that a subject may correct his intrinsic evaluation of a time interval refers to "perception awareness." Some individuals may be aware of their time perception bias and correct their estimates accordingly. Even though we cannot address this phenomenon with the data collected, we believe that a better understanding of the differences between corrected and uncorrected time perception is directly relevant to decision-making. In a follow-up research (Brocas et al., 2018) we investigate the issue of awareness of time perception biases. We show in an incentivized experiment that subjects are aware of their biases, but only partially, and they do not fully correct for them when producing time. This explains why people are predictably and systematically late despite some degree of knowledge of this tendency, and why they tend to correct for it (but not to its full extent) when it matters most. For the present paper, it suggests that the correlation highlighted in the text is likely to be an underestimate of the correlation between discounting and "true" perception.

Appendix. Supplementary material

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.geb.2018.06.007.

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