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Risk Aversion in a Dynamic Asset Allocation Experiment

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Abstract

We conduct a controlled laboratory experiment in the spirit of Merton (1971), in which subjects dynamically choose their portfolio allocation between a risk-free and risky asset. Using the optimal allocation of an investor with hyperbolic absolute risk aversion (HARA) utility, we fit the experimental choices to characterize the risk profile of our participants. Despite substantial heterogeneity, decreasing absolute risk aversion and increasing relative risk aversion are the predominant types. We also find some evidence of increased risk taking after a gain. Finally, the session level risk attitudes show a different profile than the individual descriptions of risk attitudes.

I. Introduction

Economic models, in general, and particularly finance models, often start with the assumption that the optimal decisions of a representative agent are a good description of the function of the economy in the aggregate. Even if the model considers multiple agents with different preferences, the preferences are assumed to belong to a narrow class and differ on the value of a single parameter. In any case, the choice of utility function of the representative agent or class of utility functions, in the case of multiple agents, is typically justified on the grounds of tractability.

A large body of work has studied individual behavior and proposed utility representations to use in models. In macroeconomics and finance, the typical

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methodology is to postulate a specific type of utility, derive some predictions resulting from partial or general equilibrium considerations based on that utility (or class of utilities) and, finally, empirically test the predictions.

A parallel line of analysis of this problem has been undertaken in the behavioral economics literature. This literature has mostly relied on a different methodology, namely, laboratory experiments. More precisely, this literature studies individual decision-making in narrowly defined situations. The evidence collected permits the derivation by induction of the properties a utility function should display, or even piecing together of a functional form, for example, the value function of prospect theory (Kahneman and Tversky (1979)).

Over the last two decades, other studies related to the characteristics of preferences have used this experimental methodology, and we later review the main contributions. In this paper, we propose directly estimating individual utility in an experimental setting. We elicit the functional form that best represents the decisions of participants. However, for this approach to be practical, we must select a class of parametric utility functions that can be fitted to the data. For reasons explained below, we work with the class of hyperbolic absolute risk aversion (HARA) utility functions studied in Merton (1971).

The seminal work of Merton (1971) considers the class of HARA utility functions over intertemporal consumption or final wealth in order to study the problem of portfolio optimization of a risk averse individual investor. The HARA class is broad and nests utility functions that are used not only in the portfolio optimization literature but also in most of the asset pricing, corporate finance, and macroeconomics literature. In particular, the constant relative risk aversion (CRRA) and constant absolute risk aversion (CARA) utilities are special cases of HARA.

Based on the previous considerations, we conduct an experiment that replicates Merton's (1971) setting within the technical limitations of our laboratory environment. Subjects choose how to invest their wealth between a safe and a risky asset over the course of 15 investment paths. The subject (she) starts a path with an initial endowment, which she allocates between the assets. After observing the returns of the assets, she reallocates her wealth, and new returns are observed. This dynamic process lasts for 10 periods, after which her final payoff is recorded, and a new path is started with the same initial endowment as the previous path. Overall, each subject makes 150 investment decisions with different levels of wealth.

We must emphasize that the HARA class does not include all types of utilities discussed in the literature. In particular, the HARA utility assumes risk-aversion, while the value function of Prospect Theory allows risk-loving, which can explain decisions observed both in experimental settings and in practical situations inconsistent with HARA utilities. However, we observe a low level of risk-loving behavior in our data (in less than 11% of the subjects). Other types of preferences, such as habit formation (Campbell and Cochrane (1999)) or recursive preferences (Epstein and Zin (1989)), are also not included in the HARA class. On the other hand, Merton (1971) provides a tractable dynamic setting that is suitable for an experimental setting. This allows us to assess whether many of the utilities used in economic models, namely, CRRA and CARA, are consistent with individual decisions, as well as whether other utilities within the HARA class provide a better representation. Another important observation is that HARA utilities do not aggregate in general. That is, even if we could corroborate that all economic agents display HARA preferences, we would not be able to conclude that it is possible to construct a representative agent, with the exception being if all agents belong to some narrow subset (e.g., CRRA). Our experiment provides a tool that characterizes a broader set of risk attitudes, which in turn can be used as a guide to determine the preferences (if any) of the representative agent.

This experimental setting allows us to address the following specific questions. First, consistent with the experimental literature on preferences, do we observe substantial heterogeneity in the risk attitudes of our subjects? Second, can we fit the data well using a structural estimation of an expected HARA utility model? If so, what type of HARA utility best explains the investment strategy of each participant? Third and related, do we observe frequent and/or severe deviations from neoclassical theory (i.e., systematic biases at odds with standard expected utility theory)? Fourth, if we analyze the data at the session level, how does the group behavior compare to that of subjects considered individually?

Our starting point is the optimal portfolio allocation of an expected HARA utility maximizer, as derived in Merton (1971). Given this analytical characterization, we estimate the absolute and relative risk aversion parameters of our subjects using the 150 choices made in the experiment.

Our main findings are as follows: Consistent with the existing literature, our experimental subjects (undergraduate students) are highly heterogeneous. At the same time, some risk attitudes are more prevalent than others. Most individuals increase the total amount of wealth invested in the risky asset as their wealth increases (decreasing absolute risk aversion (DARA)). They also decrease the fraction of wealth invested in the risky asset as their wealth increases (increasing relative risk aversion (IRRA)). Overall, more than half of our subjects can be confidently classified in the combined DARA–IRRA category, the risk attitude conjectured by Arrow (1971) to be the most natural among investors.

We also find some evidence of biases that is inconsistent with the assumptions of standard expected utility theory. Some subjects (19%) change their risk-taking behavior over time. More significantly, 44% of subjects exhibit a gain/loss asymmetry. Of these, the vast majority (39%) take more risks after a gain, while only 5% take more risks after a loss. Overall, many subjects exhibit some type of anomaly relative to the standard expected utility theory. However, these are small in magnitude, which is why the expected utility model performs well despite their presence.

Finally, we conduct a session level analysis using two different methodologies. We find that some types (notably, CARA) are not present at the session level, even though there are such individuals among our subjects.

Also, the relative risk aversion coefficient estimated for the sessions is typically lower than those of individuals. Therefore, while the risk attitudes of most individuals are best captured by DARA–IRRA, many of the aggregate parameters are consistent with the DARA–DRRA or DARA–CRRA types. This result occurs because DRRA agents accumulate, on average, more wealth than IRRA agents and therefore end up having a greater impact in the session.

Before proceeding to the analysis, we present a brief literature review. Methods to elicit risk attitudes in static settings abound in economics. Perhaps the best-known and most widely employed technique is the "list method" proposed by Holt and Laury (HL) (2002). This method is fast, intuitive and easy to implement. The list method offers an excellent and simple measure to compare risk attitudes across individuals and has been extended in several directions, either to improve the precision of estimates (Andersen, Harrison, Lau, and Rutström (2006), Maier and Rüger (2012)) or to obtain a more efficient algorithm (Wang, Filiba, and Camerer (2010)).[1]

However, simplicity comes at the expense of a design that is not intended (and therefore not suitable) to provide a *precise* measure of the risk preference of individuals endowed with a general utility function. For example, the HL procedure assumes CRRA utility; therefore, by construction, it cannot assess the changes in the percentage of risk taking as a function of wealth. The HL procedure also provides only interval estimates of the parameter, so it is difficult to assess the fit of the data according to the utility specification and to challenge the model.

Risk attitudes have also been explored in dynamic settings, most notably in the game show "Deal or No Deal." Assuming a CRRA functional form, Post, Van den Assem, Baltussen, and Thaler (2008) find that the expected utility theory cannot explain the contestants' decisions well and point out that previous outcomes play a significant role in the choices of participants. Andersen, Harrison, Lau, and Rutström (2008) perform a laboratory replication of "Deal or No Deal." They estimate average risk preferences without constraining the utility model to a single parameter and find moderate levels of risk aversion, with evidence suggesting IRRA. In Rapoport (1984) and Rapoport, Zwick, and Funk (1988), subjects invest in risky securities and a safe asset in a dynamic setting, and evidence is found in favor of IRRA and against CARA and CRRA. Recently, Levy and Levy (2017) showed that CRRA may be a good approximation for decisions facing large (albeit hypothetical) stakes. However, their experimental design does not allow for periodic portfolio revision.

Our methodology has a number of advantages over previous experimental designs. First, we can structurally estimate an asset allocation model based on a rich class of utility functions for each *individual* subject. We can also determine the loss in predictive and explanatory power when we restrict our analysis to simpler utility functions. Second, we can measure standard errors of individual estimates and assess the fit of the data. We can also study the structure of the noise and its relationship with wealth levels. Third, our dynamic framework is useful for measuring behavioral anomalies due to repeated exposure to risk. We can detect

[1]Other, almost equally simple, risk elicitation designs have been proposed by Becker, DeGroot, and Marschak (1964), Binswanger (1980), Hey and Orme (1994), Gneezy and Potters (1997), Eckel and Grossman (2008), and Sokol-Hessner et al. (2009), among others. For surveys of empirical and experimental elicitation procedures and results, we refer to Harrison and Rutström (2008), Charness, Gneezy, and Imas (2013), and Friedman, Isaac, James, and Sunder (2014).

any gain/loss asymmetry in behavior and determine whether a subject changes her risk attitude over the course of the experiment.

Given the investment nature of our task, our paper also relates to market experiments in which most of such tasks are implemented.[2] Levy (1994) proposes a nonstructural analysis to study risk attitudes in a market experiment. As in our paper, his results overwhelmingly support DARA but, unlike us, he does not find evidence in support of IRRA. Contrary to this literature, our main goal is to isolate risk attitudes, which is why we opt for an individual decision-making, rather than a market, set up.[3] We also provide complete information about the design to prevent subjects from forming beliefs we could not observe.[4]

Finally, our results on path dependence of choices and gain/loss asymmetry are related to the literature that highlights behavioral anomalies in choice under uncertainty.[5] Our design is not intended to test for specific behavioral anomalies nor to fit behavioral models. However, consistent with Thaler and Johnson (1990), we find that prior gains (losses) decrease (increase) risk aversion for many of our subjects.[6]

This paper is organized as follows: In Section II, we present the theoretical framework. In Section III, we describe the experimental setting. In Section IV, we present the econometric model and results of the classification analysis and estimation. In Section V, we investigate behavioral anomalies. In Section VI, we provide an aggregate analysis of the data. In Section VII, we offer some concluding remarks. An analysis of the explanatory and predictive power of our expected utility model is relegated to Appendices B and C of the Supplementary Material.

II. Theory

Consider a continuous-time setting with a risk-free security that pays a constant interest rate, and a single risky security whose price satisfies a geometric

[2]Market experiments have been extensively used in finance research to analyze asset bubbles (Smith, Suchanek, and Williams (1988), Haruvy and Noussair (2006)), to test the predictions of asset pricing models (Plott and Sunder (1988), Bossaerts and Plott (2004), Bossaerts, Plott, and Zame (2007)) and to test investor behavior (Bossaerts, Ghirardato, Guarnaschelli, and Zame (2010), Frydman, Barberis, Camerer, Bossaerts, and Rangel (2014)) among other subjects.

[3]Other related individual asset allocation experiments test whether subjects allocate portfolios efficiently (Kroll, Levy, and Rapoport (1988), Kroll and Levy (1992), and Sundali and Guerrero (2009)).

[4]Indeed, we spend substantial effort during the instruction period to explain, in detail, the financial environment of the experiment so that expectations play as small of a role as possible (for a survey on the rapidly expanding experimental literature studying the effect of expectations on risk taking behavior in macroeconomics and finance, we refer to Assenza, Bao, Hommes, and Massaro (2014)).

[5]Discrepancies between observed behavior and theoretical predictions may come from errors in choices (Jacobson and Petrie (2009)), reference dependent preferences (Kőszegi and Rabin (2006), Abeler, Falk, Goette, and Huffman (2011), Knetsch and Wong (2009), Ericson and Fuster (2011), and Sokol-Hessner et al. (2009)), or disappointment aversion (Choi, Fisman, Gale, and Kariv (2007), Gill and Prowse (2012)) among other reasons.

[6]The way prior outcomes affect subsequent risk taking is not a settled matter. See Imas (2016) for a summary and set of experiments showing how different types of losses (paper vs. realized) produce different risk choices immediately after.

Brownian motion process. Merton (1971) shows that, for the class of HARA utility, the optimal investment policy of the economic agent has an explicit solution.[7]

At each instant t, an agent (she) allocates her wealth $X(t)$ between 2 assets, a risky asset A and safe asset B. At $t=0$, her initial wealth is $X(0)=x_0>0$. The temporal horizon is finite and equal to T. The agent can reallocate her portfolio at each instant t until date T, which is the time at which she enjoys her accumulated wealth $X(T)$. Therefore, at each t, she maximizes the expected utility of wealth at time T. We assume that the agent's preferences are characterized by the general HARA utility function with the 2 parameters, γ and η, first used by Merton (1971) in a dynamic portfolio allocation.

Formally:

$$(1) \qquad U(X) = \frac{1-\gamma}{\gamma}\left(\frac{X}{1-\gamma}+\eta\right)^{\gamma}$$

with the following restrictions:

$$\gamma \neq 1, \quad \frac{X}{1-\gamma}+\eta > 0 \quad \text{and} \quad \eta = 1 \quad \text{if } \gamma = -\infty.$$

This family of utility functions is rich in the sense that it encompasses utility functions with absolute and relative risk aversion that are increasing, constant or decreasing, depending on γ and η.[8] The agent exhibits decreasing absolute risk aversion when $-\infty<\gamma<1$ and constant absolute risk aversion when $\gamma\to+\infty$ or $\gamma\to-\infty$. She exhibits increasing, constant and decreasing relative risk aversion when $\eta>0$, $\eta=0$ and $\eta<0$, respectively.

The price of the safe asset $B(t)$ evolves as follows:

$$(2) \qquad dB(t) = rB(t)dt,$$

where $r>0$. The price of the risky asset $A(t)$ follows a geometric Brownian motion process with drift $\mu(>r)$ and diffusion σ (>0). Formally:

$$(3) \qquad dA(t) = \mu A(t)dt+\sigma A(t)dW(t),$$

where $W(t)$ is a standard Brownian motion process. Let $\pi(t)$ be the amount of wealth allocated to the risky asset A at date t. The wealth $X(t)$ grows as follows:

$$\begin{aligned} dX(t) &= \pi(t)\mu dt+\pi(t)\sigma dW(t)+[X(t)-\pi(t)]rdt \\ &= [X(t)r+\pi(t)(\mu-r)]dt+\pi(t)\sigma dW(t). \end{aligned}$$

At each date t, the agent solves the following problem $\mathcal{P}$:

$$\begin{aligned} \mathcal{P}: \quad & \max_{\pi} E\left[U(X(T))\right] \\ \text{s.t} \quad dX(t) &= [X(t)r+\pi(\mu-r)]dt+\pi(t)\sigma dW(t), \\ X(0) &= x_o. \end{aligned}$$

[7]The previous setting also amounts to dynamic completeness. This notion is studied in an experimental setting by Bossaerts, Meloso, and Zame (2008).

[8]A more general specification of the HARA utility function is: $U(X)=\frac{1-\gamma}{\gamma}\left(\frac{\beta X}{1-\gamma}+\eta\right)^{\gamma}$. In our case, the parameter β is not identified and cannot be estimated.

Given the complete markets assumption and specification of utility and asset returns, our problem has a closed-form solution that we summarize in the next result.

Proposition 1. If markets are complete and time is continuous, the optimal amount allocated to the risky asset at date t when the accumulated wealth is $X(t)$ is:

$$\hat{\pi}(t) \quad = \quad \frac{\mu - r}{\sigma^2}\left(\frac{X(t)}{1-\gamma} + \eta e^{-r(T-t)}\right). \tag{4}$$

Proof. It is straightforward from the by now standard martingale representation methodology of Karatzas, Lehoczky, and Shreve (1987) and Cox and Huang (1989).

The amount allocated to the risky asset depends on the current wealth $X(t)$, the investment horizon left at $T-t$, the parameters that characterize the return dynamics of the assets, and the risk aversion parameters. The model predicts that the amount allocated to the risky asset increases in the current wealth if the agent exhibits decreasing absolute risk aversion ($\gamma < 1$). Also, the allocation depends on current wealth irrespective of how wealth has been accumulated in the past. Finally, when $\eta > 0$ (respectively, $\eta < 0$), $\hat{\pi}(t)$ increases (respectively, decreases) as time passes. Note that $\eta = 0$ corresponds to the CRRA specification, where the agent invests a constant proportion of her wealth in the risky asset irrespective of the level of wealth and the horizon left to invest.

The risk attitude of each agent is characterized by 2 dimensions; absolute risk aversion (ARA) and relative risk aversion (RRA), which can each be increasing (I), constant (C) or decreasing (D) in wealth. These dimensions are determined by the (γ, η) parameter combination of the individual, which we call "type" (Table 1). Equation (4) predicts each type in terms of the amount of wealth invested in the risky asset.

TABLE 1

Risk Types as a Function of Risk Aversion Parameters

Table 1 summarizes the risk types as a function of η and γ, given the parametric restrictions in the utility function. It reports which types are likely to be constrained and for which wealth levels (indicated by [a] and [b]) and also reports how investment varies with wealth. D(C)(I)RRA denotes Decreasing (Constant) (Increasing) Relative Risk Aversion; D(C)(I)ARA denotes Decreasing (Constant) (Increasing) Absolute Risk Aversion. [a] $\hat{\pi}=0$ for small X and $\hat{\pi}=X$ for large X; [b] $\hat{\pi}=X$ for small X and $\hat{\pi}=0$ for large X.

η Value	$\gamma<1$	$\gamma>1$	$\gamma=-\infty$	$\gamma=+\infty$
$\eta<0$	DARA–DRRA[a]	—	—	—
$\eta=0$	DARA–CRRA	—	—	—
$\eta>0$	DARA–IRRA[b]	IARA–IRRA[b]	CARA–IRRA[b]	CARA–IRRA[b]
	$\partial\hat{\pi}/\partial X>0$	$\partial\hat{\pi}/\partial X<0$	$\partial\hat{\pi}/\partial X=0$	$\partial\hat{\pi}/\partial X=0$

First, all types with DARA increase the risky investment as wealth increases. Of these, an agent with decreasing relative risk aversion (DARA–DRRA type) is willing to short-sell when her wealth is low ($\hat{\pi}(t) < 0$ when $X(t)$ is small). By contrast, an agent with increasing relative risk aversion (DARA–IRRA type) is willing to borrow when her wealth is low ($\hat{\pi}(t) > X(t)$ when $X(t)$ is small).

Second, types with IARA decrease the risky investment as wealth increases. Of these, an agent with increasing relative risk aversion (IARA–IRRA type) will invest a positive amount of wealth in the risky asset only when her wealth is low.

The closed-form solution for the optimal portfolio requires complete markets. Complete markets allow investors to borrow and take short positions in the risky security which, given the nature of the experiment, we must rule out. Therefore, our results are an approximation. When we present our results, we will discuss the impact of these restrictions. Nevertheless, some of the qualitative properties of the solution are not affected.

In particular, agents represented by DARA, CARA and IARA utility functions will, respectively, choose to invest (weakly) more, the same and (weakly) less total amounts in the risky asset as their wealth increases (Table 1). Similarly, agents represented by DRRA, CRRA and IRRA utility functions will, respectively, choose to invest a (weakly) larger, equal and (weakly) smaller fraction of their wealth in the risky asset as their wealth increases.

III. Experimental Design

The main objective of this paper is to study the dynamic portfolio choice of agents in a controlled laboratory setting. To this purpose, we design a dynamic investment problem that follows as closely as technically feasible the setting of the theory section. Subjects in the experiment allocate wealth between one safe and one risky asset during 15 investments *paths* consisting of 10 *periods* each.

The experiment consists of 13 sessions run in the Los Angeles Behavioral Economics Laboratory (LABEL) at the University of Southern California.[9] Each session has between 7 and 10 subjects for a total of 120 recruited subjects, of which 3 are omitted from the analysis due to software malfunction. All subjects participate in three treatments that are always performed in the same order. The first treatment corresponds to the paradigm under study in this paper. The results of the other two treatments are reported in Brocas, Carrillo, Giga, and Zapatero (2019).

Each subject starts each path in period 1 with an endowment of \$3, which she allocates between 2 assets: a risky asset A and safe asset B. After period 1 ends, each subject earns a return on her portfolio and moves to period 2. She then reallocates her portfolio and earns new returns. This process continues for a total of 10 periods. After period 10, the investment path ends and the subject's final payoff in that path is recorded. Each subject then moves to the next investment path, where her endowment is reset to \$3. Subjects have 10 seconds to make their decision in period 1 of each path and 6 seconds in periods 2 to 10. They all begin and end their investment paths at the same time. All subjects go through 15 paths for a total of 150 choices. Subjects know at the beginning of the experiment the number of paths and periods in each path they will go through.

The return of the safe asset B is 3%, while the return of asset A is drawn from a lognormal distribution with a mean of 23.5% and standard deviation of 73.4%.[10]

[9]For information about the laboratory, please visit http://dornsife.usc.edu/label.

[10]This (unrealistically high) mean and standard deviation ensure enough volatility in returns for generating interesting wealth effects and comparative statics. In the discrete version of the experiment,

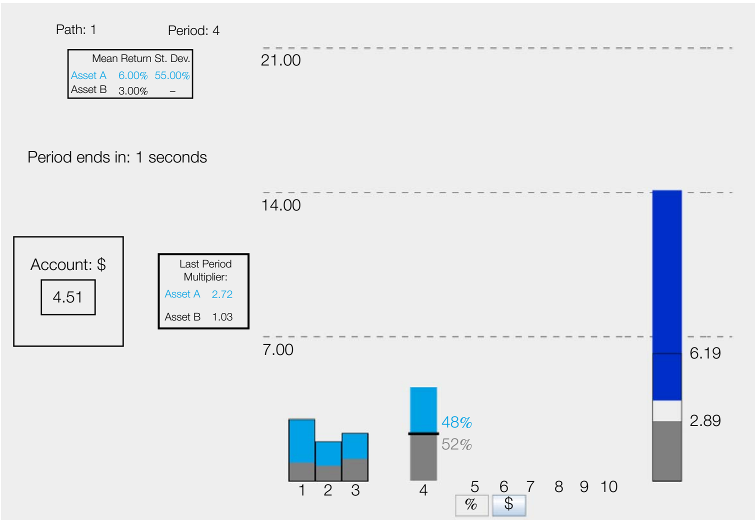
The parameters do not change throughout the experiment. The draw of the return is presented in the form of a multiplier, that is, the number that multiplies the allocation to that asset. Importantly, all participants in a session are subject to the same draws, which makes it possible to analyze the aggregate portfolio of each session (see Section VI). At the same time, we make clear to each subject that her return is in no way affected by the allocation decision of the other subject.

Figure 1 provides a screenshot describing what a subject sees in a given period of a path. Current wealth is represented by the vertical bar above the current

FIGURE 1

Screenshot of Path 1: Period 4

Figure 1 provides a screenshot that shows what a participant sees in the computer screen in a given period of a path; period 4 in this example. It includes total wealth, values of the last multipliers and bars that represent a breakdown of the history of wealth up to now and expected final wealth with the current allocation.



the evolution of wealth is:

$$X(t+1) \quad = \quad X_B(t)(1+r)+X_A(t)e^R,$$

where $X_i(t)$ is the dollar amount invested in asset $i \in \{A, B\}$ and R is normally distributed with a mean of 0.06 and standard deviation of 0.55. Note that the return of the risky asset, e^R, is lognormally distributed, thus the worst case for the subject is to lose her investment in asset A. In a part of the instructions and on the upper left corner of the screen, we described in words the parameters of the return on asset A as being normally distributed with a mean of 6% and standard deviation of 55%, when it should have read lognormally distributed with a mean of 23.5% and a standard deviation of 73.4%. In other words, we accidentally described R instead of e^R. Nevertheless, we are confident that this did not impact the results, as the rest of the instructions and accompanying slides vividly and correctly describe the entire distribution of asset A through graphical and video examples. Moreover, students are shown a correctly specified interactive projection bar at the end of the screen that informs them of the possible distribution of payoffs at the end of the path as they change their current allocation. Previous research shows the importance of visual and interactive tools for financial literacy (Lusardi et al. (2017)). Lastly, students had five practice paths and a quiz before starting their paid trials, which was enough to experiment with the bar and the payoffs.

period number (period 4 in this example). Initially, the bar is not active and wealth is not allocated to either asset. Subjects must click on the bar to activate wealth and move a horizontal slider to divide their wealth between assets A and B. The upper portion of the bar represents the money invested in risky asset A and the lower portion represents the money invested in asset B. The figures on the right side of the bar show the allocation. To facilitate her reasoning, each subject may change the display of the allocation at any time between the percentage invested in each asset (box labeled "%") and the total amount in each asset (box labeled "$"). After the period expires, returns are applied and subjects move to the next period. A new bar with a height corresponding to the new wealth appears to the right of the previous period for the new period and becomes inactive again. Subjects must reactivate it to choose a new allocation; otherwise, they earn no interest in that period and their account simply carries over.[11] Subjects observe bars to the left of the current period that remind them of their past allocations and returns. These bars accumulate up to period 10 and are then reset for the new path.

Finally, the left-hand side of the screen shows a summary of the information of the main ingredients of the experiment: i) the current path and period; ii) a reminder of the mean and standard deviation of returns of assets A and B; iii) the time remaining to make a choice in the current period; iv) the accumulated wealth in the current path; and v) the multiplier of assets A and B in the last period of the current path.

This dynamic wealth allocation problem is challenging and may require substantial learning. We develop a highly illustrative 40-minute instruction period using numerical examples, videos, five practice paths and a comprehension quiz (instructions can be found in Appendix A of the Supplementary Material). To help with the cognitive strain, we also add a projection bar to the right side of the screen that tells the subject what she would expect if she were to keep her current investment strategy until the last period. The bar shows the potential accumulated earnings from asset B and identifies the 20th, 50th and 80th percentile of the earning distribution from asset A (see Figure 1). As the participant changes her allocation, the projection bar automatically adjusts.[12]

Each participant received a $5 show-up fee and her final earnings in the final period of *one* randomly selected path (the average earnings were $9.5 with a maximum of $41).[13] At the end of the experiment, we collected answers on education, demographic and income related questions as well as participants' own description of the strategies employed. The length of the experiment, including all three treatments and the survey, was 2 hours.

Note that the experimental design closely follows the theory with two important differences, both of which were introduced for technical reasons. First, choices are made in discrete time, with only 10 decisions per path. Continuous time is difficult to implement in an experimental setting (although not impossible,

[11]This helps prevent subjects' inertia and a bias towards any status quo allocation. The level of inactivity in our experiment was negligible.

[12]We carefully explain the function of the bar by simulating a large number of period-by-period trajectories of wealth coming from a given allocation strategy.

[13]Subjects were also compensated for the other two treatments. The total compensation in the experiment averaged $23, with a maximum of $244.

see, e.g., Friedman and Oprea (2012)).[14] Second, we do not allow our participants to borrow or short-sell, which means that the markets are incomplete in the sense of Merton (1971). Borrowing and short-selling are difficult to implement experimentally since they may result in taking money away from participants. Our data analysis takes this restriction into account.

IV. Results

Our first objective is to test how well the expected utility theory fits the data. We adopt a structural approach and estimate the risk parameters (γ, η) of each subject, assuming they behave according to the expected utility theory model. This approach is used to classify our subjects according to their risk type.

A. Econometric Model

According to equation (4) and subject to the above-mentioned caveats of incomplete markets and discrete time, the expected utility theory predicts that the portfolio allocation and wealth will vary over time according to the following system:

$$\begin{cases} \hat{\pi}(t) = \dfrac{\mu - r}{\sigma^2}\left(\dfrac{X(t)}{1-\gamma} + \eta\, e^{-r(T-t)}\right), \\ dX(t) = \left[X(t)r + \hat{\pi}(t)(\mu - r)\right]dt + \hat{\pi}(t)\sigma\, dW(t). \end{cases}$$

The parameters γ and η can be estimated from equation (1) using least squares fitting. Since our data are obtained in discrete time, we consider the discrete version of the model. For each individual, in each path i and at each period t, we observe the current wealth $X_{i,t}$ and the chosen allocation of this wealth to the risky asset $\pi_{i,t}$. Let $F_t = e^{-r(T-t)}$, our structural econometric model given HARA utility is $\mathcal{M}^{\text{HARA}}$:

$$\pi_{i,t} \quad = \quad aX_{i,t} + bF_t + u_{i,t}, \tag{5}$$

where $a = \frac{\mu - r}{\sigma^2(1-\gamma)}$, $b = \frac{(\mu - r)\eta}{\sigma^2}$ and $u_{i,t} \sim \mathcal{N}(0, \sigma_u^2)$ is an error term.[15] Given a and b, the parameters γ and η are identified. In Section IV.C, we classify the risk attitude of our subjects by fitting this model to their decisions.

Note that a myopic decision-maker would maximize the instantaneous expected utility $E[U(X(t))]$ at each period t. This problem has a simple closed-form solution: The optimal allocation in the risky asset is obtained by replacing $e^{-r(T-t)}$ with 1 in the equilibrium equation of Proposition 1. For our data, $e^{-r(T-t)} \in [0.7, 1]$. This value is close enough to 1 to make the myopic model very similar to the forward-looking model.[16]

[14] Duffie and Protter (1992) provide a theoretical discussion on the convergence of discrete-time processes to continuous-time processes.

[15] We relax the assumptions on the error term's distribution later (see Section IV.C.1). For robustness purposes, we evaluate the data from unconstrained subjects using the beta regression model. We also test the sensitivity of our risk elicitation to the subject's perception of the return parameters. In both analyses, available from the authors upon request, the results remain qualitatively the same.

[16] We conducted the analysis based on the myopic model and did not find any qualitative changes in the classification of our subjects.

Also, we require enough variation in wealth within subjects for an accurate estimation. In half of the sample, the 5th and 95th percentiles of wealth are approximately \$1 and \$15, respectively. For the other half of the sample, the range extends from \$1 to \$20, respectively. Although these figures are not excessively large, the dispersion is important enough to obtain reliable estimates of absolute and relative risk aversion.

Lastly, our structural model is well specified only if subjects do not systematically invest all their wealth in the safe or the risky asset, which poses a challenge. On one hand, treating the data as if all choices are interior biases the interpretation of the parameters and the residuals of the regression. On the other hand, eliminating the constrained choices from the analysis also biases the estimated parameters. The solution we propose is to separately classify subjects who hit the bounds often from those who do not.

B. Classification Criteria: Constrained versus Unconstrained Subjects

Our first task is to empirically determine which subjects are affected by the inability to short-sell (i.e., to set $\pi_t < 0$) and/or borrow (i.e., to set $\pi_t > X_t$). For the large majority of our subjects, the pressure to short-sell or borrow is low. At the aggregate level, subjects invest all their wealth in the safe asset 2.2% of the time and in the risky asset 8.2% of the time.[17] At the individual level, there is heterogeneity in behavior (Table 2).

Only 34 subjects never hit a constraint. However, if we combine these subjects with those who hit the constraints no more than 10% of the time, we account for 81 individuals, or 69% of the sample. We call these subjects "unconstrained." Of the remaining subjects, 11 would have liked to borrow and 25 would have liked to both borrow and short-sell. We call these subjects "constrained."

TABLE 2
Number of Subjects Affected by the Constraints

Table 2 reports the distribution (% of trials) of subjects as a function of their likelihood to hit the constraints, as described at the beginning of Section IV.B.

Type of Constraint	(0%, 10%]	(10%, 20%]	(20%, 100%]	Total
Hit $\pi_t = 0$ only	10	0	0	10
Hit $\pi_t = X_t$ only	24	3	8	35
Hit $\pi_t \in \{0, X_t\}$	13	11	14	38
$\pi_t \in (0, X_t)$ always	n/a	n/a	n/a	34

C. Estimation and Classification

1. Unconstrained Subjects

We estimate the risk aversion parameters (γ, η) of the 81 unconstrained subjects for which the econometric model $\mathcal{M}^{\text{HARA}}$ is well specified. Given that our observations are repeated measures for the same subject and that wealth follows a stochastic process, we must be careful about issues that arise naturally in this time

[17] A choice is defined as nonconstrained (interior) when the allocation to the risky asset is greater than 2% and smaller than 98% of the wealth.

series framework and that may contradict the underlying assumptions required for the use of the least squares method.

First, the error term should have a constant variance. We run a standard OLS on each individual's data set and apply the White test to detect the presence of heteroscedasticity. We find that the variance of the residuals increases with the level of wealth for 73 out of the 81 unconstrained subjects (at the 5% significance level) and is constant for the rest.

Second, error terms should be uncorrelated across periods. We test for the serial correlation for each participant by looking at the residuals of the OLS regression, denoted by $\hat{u}_{i,t}$. Note first that an error at period $t-1$ applied to the amount invested in the risky asset at that period affects the wealth level at period t. Therefore, regressors are not independent of the error term. To account for this, we use the Breusch–Godfrey test, which allows explanatory variables to not be strictly exogenous. Formally, we consider the regression:

$$\hat{u}_{i,t} \quad = \quad \beta_0 + \beta_1 X_{i,t} + \beta_2 F_t + \rho \hat{u}_{i,t-1} + v_{i,t},$$

where $X_{i,t}$ and F_t account for weak exogeneity and $v_{i,t}$ are assumed to be i.i.d. with a normal distribution $\mathcal{N}(0, \sigma_v^2)$. We use robust standard errors in our test and find a first-order serial correlation ($\rho > 0$) for 63 out of 81 subjects. To correct for heteroscedasticity and autocorrelation, we run the OLS regression with Newey–West standard errors.

Figure 2 and Table 3 present the estimates of the value of the parameters γ and η, as well as the type of risk aversion they correspond to. We observe

FIGURE 2

Estimated Parameters of the 81 Unconstrained Subjects

Figure 2 displays the estimated (γ, η) risk parameters of the 81 unconstrained subjects using the structural model $\mathcal{M}^{\text{HARA}}$ presented in equation (5). D(C)(I)-R denotes Decreasing (Constant) (Increasing) Relative Risk Aversion; D(C)(I)-A denotes Decreasing (Constant) (Increasing) Absolute Risk Aversion.

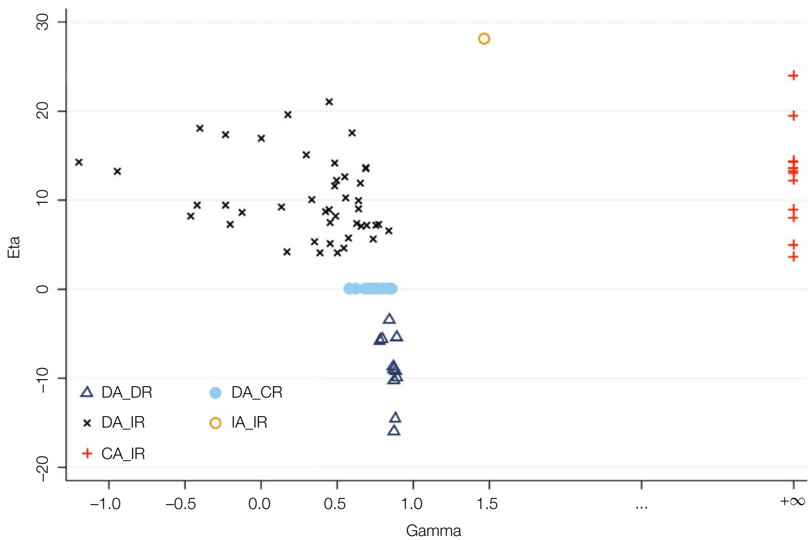


TABLE 3
Risk Attitude of the Unconstrained Subjects

Table 3 reports the relative and absolute risk aversion attitudes based on the estimated parameters. D(C)(I)RRA denotes Decreasing (Constant) (Increasing) Relative Risk Aversion; D(C)(I)ARA denotes Decreasing (Constant) (Increasing) Absolute Risk Aversion.

Risk Attitude	No. of Subjects
DARA–DRRA	11
DARA–CRRA	13
DARA–IRRA	44
IARA–IRRA	1
CARA–IRRA	12
Total	81

substantial heterogeneity in risk attitudes. At the same time, the vast majority of subjects are DARA ($\gamma < 1$ for 84% of subjects) and IRRA ($\eta > 0$ for 70% of subjects). Overall, 54% of subjects are willing to increase their total investment in the risky asset and decrease the fraction of investment in the risky asset as their wealth increases. These are the DARA–IRRA subjects ($\gamma < 1$ and $\eta > 0$) conjectured by Arrow (1971) to empirically be the most plausible types.

By contrast, the simple one parameter specifications commonly used in the literature do not capture the risk attitude of many of our subjects well: Only 15% of our subjects are CARA ($\gamma \to +\infty$) and 16% are CRRA ($\eta = 0$).[18]

We next compare our results to existing estimates in the literature, such as, HL. For this, we estimate our structural model assuming the familiar functional form:

$$\tilde{U}(X) \quad = \quad \frac{X^{1-\xi}}{1-\xi}$$

used in HL. In this case, the solution to the problem $\mathcal{P}$ described in Section II is well-known. Indeed, the agent invests a constant fraction of wealth in the risky asset:

$$\hat{\pi}(t) \quad = \quad \frac{1}{\xi}\frac{\mu - r}{\sigma^2}X(t).$$

Analogously to our strategy in Section IV.A, we estimate ξ from the following econometric model:

$$\text{(6)} \qquad \pi_{i,t} \quad = \quad cX_{i,t} + \nu_{i,t},$$

where $c = \frac{\mu - r}{\xi\sigma^2}$ and $\nu_{i,t} \sim \mathcal{N}(0, \sigma_\nu^2)$ is an error term. We call this model $\mathcal{M}^{\text{CRRA}}$ and compare our estimates of ξ to those in HL. Because of the way the experiment is designed, HL only gives range estimates for the parameter ξ. Table 4 reports the proportion of subjects who fall into each range of ξ in our model ($\mathcal{M}^{\text{CRRA}}$), as well as in the low stakes (\$0.10–\$3.85, HL-low) and high stakes (\$2–\$77, HL-high) treatments of HL.

Our estimates are substantially more concentrated than in HL. Only 11% of our subjects exhibit risk-neutrality or risk-loving preferences ($\xi < 0.15$) as opposed to 34% and 19% in HL-low and HL-high, respectively. Unlike HL, we also

[18]By $\eta = 0$ we mean that the estimated parameter is not statistically different from 0 at the 95% confidence interval. For our classification, we use CARA and CRRA as the null hypotheses which may overclassify subjects in those categories.

TABLE 4

Risk Estimates: A Comparative Analysis with Holt and Laury (2002)

Table 4 compares our estimation results (column 4) of the coefficient of relative risk aversion with those reported in Holt and Laury (2002) (columns 2 and 3). The risk elicitation methodology in Holt and Laury is one of the most widely employed techniques.

Risk Aversion	HL-Low	HL-High	$\mathcal{M}^{\text{CRRA}}$
$\xi < 0.15$	0.34	0.19	0.11
$0.15 \leq \xi < 0.41$	0.26	0.19	0.61
$0.41 \leq \xi < 0.68$	0.23	0.23	0.27
$0.68 \leq \xi < 0.97$	0.13	0.22	0.01
$0.97 \leq \xi < 1.37$	0.03	0.11	0.00
$1.37 \leq \xi$	0.01	0.06	0.00
No. of subjects	175	150	81

find no evidence of high ($0.97 \leq \xi < 1.37$) or extremely high ($1.37 \leq \xi$) risk aversion. Overall, we have twice as many subjects as HL in the expected range (88% against 49% and 42% in $0.15 \leq \xi < 0.68$). These differences are important and are partly due to differences in the design and partly due to the misspecification of the CRRA utility function in our experiment (and possibly in theirs as well). These differences highlight the advantages of a rich experimental setting to better estimate risk aversion and a 2-parameter specification to capture the heterogeneity present in the relative risk aversion of subjects.

2. Constrained Subjects

Next, we study the risk attitude of the 36 subjects who, according to the analysis in Section IV.B, are constrained by their inability to borrow and short-sell. As noted before, the tendency to invest all wealth in the safe or the risky asset should depend on the amount of wealth. We first assess how wealth affects their probability of hitting each bound. More specifically, we estimate a probit regression on the following two models:

$$\begin{aligned}\pi_{i,t}^{\max} &= b_0^{\max} + b_1^{\max} w_{i,t} + \epsilon_{i,t}^{\max},\\ \pi_{i,t}^{\min} &= b_0^{\min} + b_1^{\min} w_{i,t} + \epsilon_{i,t}^{\min},\end{aligned}$$

where $\pi_{i,t}^{\max}$ takes a value of 1 if $\pi_{i,t} = w_{i,t}$ and 0 otherwise and $\pi_{i,t}^{\min}$ takes a value of 1 if $\pi_{i,t} = 0$ and 0 otherwise. We establish an effect when $b_1^{\max}$ or $b_1^{\min}$ are different from 0 at the 5% significance level.

We find three distinct groups of individuals (Table 5). There are 28 "constrained IRRA" subjects, who invest their entire wealth in the risky asset when their wealth is low enough ($b_1^{\max} < 0$), their entire wealth in the safe asset when

TABLE 5

Risk Attitude of Constrained Subjects

Table 5, analogue to Table 3, reports the risk attitudes of subjects constrained by their inability to borrow or short-sell. I(D)RRA denotes Increasing (Decreasing) Relative Risk Aversion. DARA denotes Decreasing Absolute Risk Aversion.

Risk Attitude	No. of Subjects
Constrained IRRA	28
Constrained DARA–DRRA	1
Constrained irregulars	7
Total	36

their wealth is high enough ($b_1^{\min} > 0$), or both. This behavior is consistent with IRRA, although it can also be compatible with risk neutrality for low enough wealth levels.[19] There is 1 "constrained DARA–DRRA" subject who invests his entire wealth in the safe asset when his wealth is low enough ($b_1^{\min} < 0$) and in the risky asset when his wealth is high enough ($b_1^{\max} > 0$). This behavior is consistent only with DARA–DRRA. Finally, there are 7 "constrained irregular" subjects who exhibit an irregular and volatile behavior with no discernible patterns or statistically significant effects. The result (which is the analogue of Table 3 for the constrained subject sample) is summarized in Table 5.

Overall, as for the unconstrained subjects, there is substantial heterogeneity among the constrained subjects. A majority (78%) exhibit increasing relative risk aversion, and almost half (44%) exhibit decreasing absolute risk aversion.

In Appendix B of the Supplementary Material, we present a number of robustness checks. We show that the class of HARA utility functions explains the investment decisions of the participants well and has good predictive power in out-of-sample analysis. We find a statistically reliable relationship between the investment decision and the set of independent variables. Also, when we estimate the parameters using a subsample (either eight randomly chosen paths or the first five periods of all paths), we can predict the behavior in the complementary subsample well. Finally, if we restrict attention to CRRA utility, the accuracy of our overall estimates suffers in a statistically significant way for one-third of our subjects. We conclude that CRRA utility has appeal due to its simplicity and analytical properties. However, this might come at the cost of a bias for a sizeable proportion of the population.

Finally, we use our questionnaire to study the correlation between risk attitude and demographics. We find an overrepresentation of males in the population of subjects who are affected by the inability to borrow (hit $\pi_t = X_t$). More precisely, 33% of males versus 18% of females are in the Constrained IRRA group. Among the unconstrained subjects, the distributions of types in the male and female populations are not significantly different.

V. Behavioral Anomalies

Several studies have reported behavioral anomalies in decision-making under risk and uncertainty. One notable anomaly is the tendency of subjects to repeat choices that have generated gains in the past and avoid choices that have generated losses in the past. In a financial setting, this tendency translates into repeating risky investments after a gain and moving wealth into safe assets after a loss, even when draws are known to be i.i.d. (Thaler and Johnson (1990)).

A second and related anomaly is a disproportionate preference to avoid losses relative to acquire gains, as in prospect theory (Kahneman and Tversky (1979)). In a financial setting, the reference point can be the current wealth or any other

[19] Of these subjects, 15 are best classified as DARA–IRRA, 10 are best classified as CARA–IRRA, and 3 are best classified as IARA–IRRA.

heuristic. From a dynamic perspective, the reference point is likely to change over time, suggesting that a certain degree of time dependence may be observed.[20]

Our goal in this section is to determine if there are systematic biases in choices due to dynamic considerations rather than to test specific models or fit specific parametric functions.

In our dynamic expected utility model, negative or positive shocks at $t-1$ affect wealth at t and therefore the investment decision at t. The risk attitude of each subject determines how she should respond to positive or negative shocks. To test whether subjects react differently after a positive and negative shock, or whether time dependence is present, we must control for any effect that emerges naturally from the model. To do so, we study the residuals of our corrected least squares regression in the 81 unconstrained subjects that are fitted with the $\mathcal{M}^{\text{HARA}}$ model. We explore their behavior as a function of the path as well as the returns obtained in the period immediately before.

A. Path Dependence

To test for path dependence, we run the following regression:

$$\hat{u}_{i,t} \quad = \quad \beta_0 + \alpha I_{i,t}^{\text{path}>8} + \beta_1 X_{i,t} + \beta_2 F_{i,t} + \rho \hat{u}_{i,t-1} + v_{i,t}, \tag{7}$$

where $I_{i,t}^{\text{path}>8}$ is a dummy variable that takes a value of 1 if the observation is from a late path (9 to 15), and 0 otherwise. The regression shows no evidence of path dependence for 63 subjects (at the 5% significance level). Among the remaining subjects, 9 exhibit a positive α-parameter, indicating more risk-taking behavior over time than predicted by the model, and 9 exhibit a negative α-parameter, indicating less risk-taking over time than predicted by the model.[21] A possible explanation is that subjects learn about their preferences over time and adapt their behavior gradually. To investigate this issue further, we run a regression with squared residuals as the dependent variable to assess whether the decisions of subjects become more precise over time. We find that among the 18 subjects with path dependency, one subject commits more mistakes over time (decreasing precision) and no subjects commit fewer mistakes over time.

Finally, we examine the α-coefficients of the subjects with a statistically significant effect. The largest positive and negative coefficients are $\alpha = 0.56$ and $\alpha = -0.51$, meaning that the error in the estimation due to path dependency is relatively small. To summarize, 22% of the individuals show statistically significant path dependency, but they go in both directions and are small in magnitude.

B. Gain/Loss Asymmetry

To check whether subjects react differently after a loss or a gain, we run the regression:

$$\hat{u}_{i,t} \quad = \quad \beta_0 + \alpha I_{i,t}^{\text{gain}} + \beta_1 X_{i,t} + \beta_2 F_{i,t} + \rho \hat{u}_{i,t-1} + v_{i,t},$$

[20] The literature usually uses status quo or lagged status quo as natural candidates for the reference point. Kőszegi and Rabin (2006) model the reference point as an expectation.

[21] The results are similar when we run the regression: $\hat{u}_{i,t} = \beta_0 + \alpha \text{PT}_{i,t} + \beta_1 X_{i,t} + \beta_2 F_{i,t} + \rho \hat{u}_{i,t-1} + v_{i,t}$, where the independent variable PT (Path) takes values from 1 to 15.

where $I_{i,t}^{\text{gain}}$ is a dummy variable that takes a value of 1 if the subject starts the period t after a gain at $t-1$ and 0 if she starts the period after a loss at $t-1$ (we use the White–Huber standard errors to account for heteroscedasticity). Our data show no reaction to previous gains or losses beyond the model prediction for 30 subjects (at the 5% significance level). Among the remaining 51 subjects, the vast majority (46 subjects) exhibit higher residuals after a gain. As in Thaler and Johnson (1990), these subjects take more risks after a gain than after a loss. The remaining 5 subjects exhibit the opposite pattern.

We then study the magnitude of the α-coefficient for subjects with a significant overreaction to previous outcomes. Among subjects who take more risks after a gain, 40 have a small overreaction ($1 or less) and 6 have a more substantial reaction (between $1 and $4). All 5 subjects who take more risks after a loss have a small coefficient: $|\alpha| < 0.48$. In summary, while many subjects (57%) exhibit excessive risk-taking after gains, the overreaction is small in magnitude, with some exceptions (7% of subjects).

C. Summary of the Behavioral Types

It is interesting to notice that the two sets of anomalies involve mostly different subjects: 6 individuals exhibit path dependence, 39 exhibit a gain/loss asymmetry, and only 12 exhibit both anomalies. Also, subjects with one or both anomalies are present in all of the risk-type categories described in Table 3. The remaining 24 subjects can be very confidently classified as expected utility maximizers.

In conclusion, anomalies are prevalent. Residual behavior can be attributed to systematic biases that are not captured by the structural model. At the same time, anomalies are spread among subjects and are small in magnitude, so we can fit the data to the expected utility model reasonably well.

VI. Session Level Risk Attitudes

To assess the group level risk attitudes, we now perform the same classification exercise as in Section IV.C, except that we conduct the analysis at the *session* level rather than at the individual level. Since we are not aware of any established methodology to perform a group level analysis in the setting of dynamic portfolio allocation, we explore two approaches.

The Per-Capita Agent. Recall that our experiment consists of 13 sessions with 7 to 10 participants each. This design permits an objective measure of aggregate wealth because all participants within a session are subject to the same shock. Instead of summing all the wealth accumulated by subjects in each period, we adopt a per-capita specification, which allows us to identify the risk preferences of the "per-capita agent."[22] Accordingly, the per-capita amount invested in the risky asset in each period represents per-capita agent's allocation decision. We then fit

[22]The per-capita agent is a description of the group risk attitudes and is not related to any well-known theoretical construct of a representative agent, which is intrinsically a market equilibrium concept. In a recent market experiment, Asparouhova, Bossaerts, Roy, and Zame (2016) find that individual decisions have little explanatory power for market prices. In other words, the preferences of the representative agent may look nothing like the preferences of the individuals they are composed of.

our structural model $\mathcal{M}^{\text{HARA}}$ to the transformed data. This approach makes the results of the individual and session analyses comparable.

The Fixed Effects Approach. We pool all the same session individual level data and apply our structural model $\mathcal{M}^{\text{HARA}}$ in equation (5) with added individual fixed effects and repeat the evaluation for all 13 sessions. The resulting coefficients a and b can then be interpreted as some average risk preferences for the session.

As in Section IV.C, we correct for heteroscedasticity and serial correlation using the Newey–West standard errors and estimate γ and η to obtain the risk types at the session level.[23] We conduct this analysis using the data from the 81 unconstrained subjects, so that we can draw a comparison between the individual and the session cases.

Regardless of the approach, all sessions exhibit DARA (Figure 3 and Table 6). In other words, and unlike the individual level analysis, there is no evidence of CARA at the session level. As for relative risk aversion, we find a reasonably similar proportion of CRRA estimates at the individual and session levels (16%–31%), but significantly more IRRA estimates at the individual or per-capita session (70%–62%) than at the fixed effect session level (31%).

Even when we only look at the DARA types, the estimates of γ are substantially more dispersed at the individual level ($\gamma \in (-1.2, 0.9)$) than at the session level ($\gamma \in (0.2, 0.9)$), regardless of whether we consider the per-capita or the fixed effects approach.

The estimates of η are, on average, higher at the individual than at the session level.[24] This result is important as it suggests that wealth effects are weaker when we aggregate information, either with the per-capita or the fixed effects approach,

TABLE 6

Session Level Risk Attitudes

Table 6, analogue to Table 3, reports the relative and absolute *session level* risk aversion attitudes based on the estimated parameters. To identify *session level* risk type, we employ two methods: *per-capita agent* and *fixed effects*. D(C)(I)RRA denotes Decreasing (Constant) (Increasing) Relative Risk Aversion; D(C)(I)ARA denotes Decreasing (Constant) (Increasing) Absolute Risk Aversion.

Risk Attitude	Per-Capita Agent	Fixed Effects
DARA–DRRA	2	5
DARA–CRRA	3	4
DARA–IRRA	8	4
IARA–IRRA	0	0
CARA–IRRA	0	0
No. of sessions	13	13

Our description of group risk attitudes is more relevant to a wealth manager who may find it useful to know the preferences of each individual client and the dollar-weighted preferences of the entire group.

[23]By aggregating wealth, the per-capita investment is always interior. The results of the White test indicate the presence of heteroscedasticity in all sessions. The Breusch–Godfrey test reveals a first-order serial correlation in 8 of the 13 sessions.

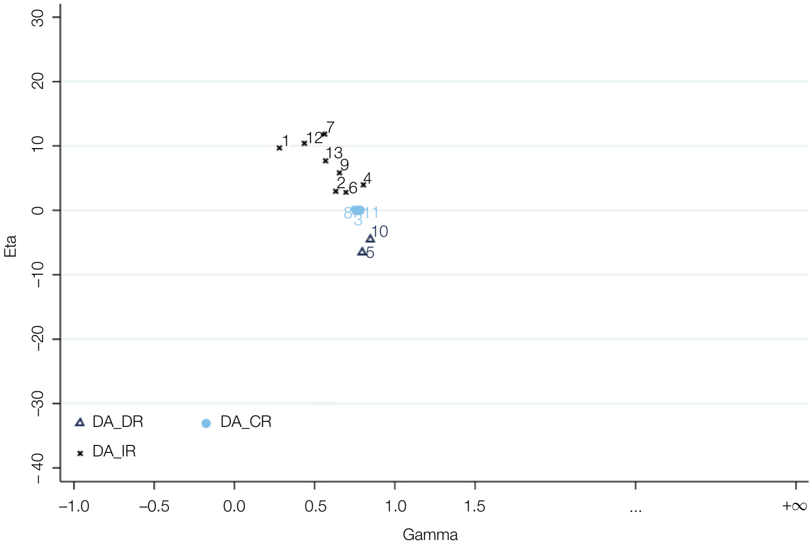
[24]The estimates of η are also more concentrated at the per-capita session than at the individual level. The comparison of dispersion is more ambiguous between the fixed effect session and individual level due to the low estimates in sessions 3 and 5.

FIGURE 3

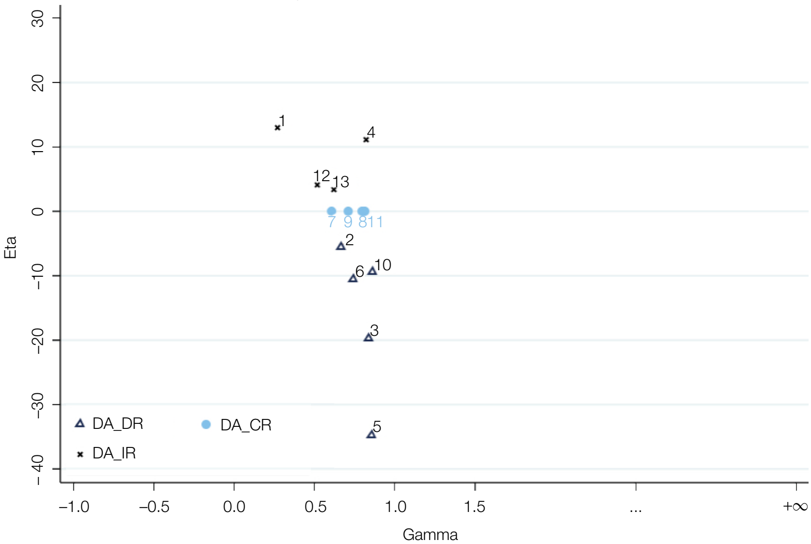
Estimated Parameters at the Session Level (81 Unconstrained Subjects)

Figure 3, the analogue of Figure 2, displays risk estimates for the session level analysis. Graph A shows the estimated (γ, η) risk parameters of the 13 per-capita agents of our experiment when the 81 unconstrained subjects are considered. Graph B shows the estimated (γ, η) risk parameters of the 13 sessions using the fixed effect approach. Sessions are labeled to facilitate comparisons of the two approaches. D(C)(I)-R denotes Decreasing (Constant) (Increasing) Relative Risk Aversion; DA denotes Decreasing Absolute Risk Aversion.

Graph A. Estimated Parameters: Per-Capita Agent



Graph B. Estimated Parameters: Fixed Effects



because DRRA agents accumulate, on average, more wealth than IRRA agents and therefore end up having more weight on the session behavior. Therefore, even though there are fewer DRRA than IRRA individuals, their impact in the economy is larger. This effect is exacerbated with the fixed effects approach, most likely due to the sensitivity of the OLS regression to outlier observations, that is, individuals who accumulate high wealth by investing heavily in the risky asset. Finally, we also estimate the risk parameters of the structural model at the level of the entire experiment with both individual and session fixed effects. We obtain $\eta^* = -8.7$ and $\gamma^* = 0.76$, suggesting that if we aggregate the entire population, the "average preferences" are best described by DARA–DRRA.

For the per-capita approach, we check for evidence of behavioral anomalies at the aggregate level by replicating the analysis of Section V. There is little evidence of path dependence; only three sessions show an effect, and the magnitudes are small. By contrast and consistent with Thaler and Johnson (1990), we find that in all 13 sessions the per-capita agent takes more risk after a gain than after a loss (that is, residuals are higher after a gain). Once again, however, the magnitude of the anomaly is small.

Finally, in Appendix C of the Supplementary Material, we perform the same robustness checks for the per-capita agent as we did for the individual analysis, and obtain similar conclusions. Most notably, the out-of-sample predictions of the HARA model significantly improve those of CRRA in one-third of the sessions and are very similar in the rest.

In summary, while the individual analysis shows that the majority of subjects are DARA–IRRA expected utility maximizers when we perform some type of aggregation, the resulting behavior moves closer to DARA–DRRA or DARA–CRRA types.

VII. Conclusion

In this paper, we report the results of an experiment in which 117 subjects dynamically choose their wealth allocation. Assuming a HARA utility function, we first construct a structural dynamic choice model, which we then use to estimate the absolute and relative risk aversion of the participants. Although technically more complex, this method has the advantage of providing more accurate estimates than traditional risk elicitation techniques.

Even though we find substantial heterogeneity in behavior, decreasing absolute risk aversion and increasing relative risk aversion are the most prevalent subtypes, and we can confidently classify more than half of subjects in the combined DARA–IRRA category. We also find evidence of increased risk taking after a gain. yet the effect is small in magnitude and the behavior of subjects is generally well accounted for by the expected utility model. Finally, our design allows us to perform an aggregate analysis. We find that the session level risk attitudes show a different profile than the individual description of risk attitudes, with lower coefficients of relative risk aversion, more concentration in the coefficients of absolute risk aversion, and no evidence of constant absolute risk aversion.

Recent papers have argued that risk attitudes are volatile and difficult to pinpoint (see Friedman et al. (2014) for a survey). Our analysis suggests that if the experimental setting is rich enough, it is possible to accurately estimate (stable) risk preferences. This result is encouraging given the paramount importance for microeconomic theory in understanding risk choices in financial, insurance and environmental settings, to name a few.

Supplementary Material

Supplementary Material for this article is available at https://doi.org/10.1017/S0022109018001151.

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