

Sequential Auctions with Capacity Constraints: an Experimental Investigation

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Abstract

We conduct a laboratory experiment where groups of 4 subjects constrained to obtain at most one good each, sequentially bid for 3 goods in first and second price auctions. Subjects learn at the beginning of each auction their valuation for the good and exit the auction once they have obtained one good. We show that, contrary to equilibrium predictions, subjects' bidding behavior is excessively similar across units and across mechanisms at the aggregate level. We provide two (complementary) explanations for these departures. One is bounded rationality. Subjects do not fully comprehend subtle differences between mechanisms. The other is self-selection. Subjects are very heterogeneous and some of them deviate more from equilibrium than others. Since deviations take mostly the form of overbidding, these subjects win the first or second good and exit the auction, leaving those who play closer to theoretical predictions to bid for the third good. Support for this hypothesis comes from the documented higher bidding, lower efficiency and lower profits associated with the first and second unit compared to the third one.

Introduction

Bidding behavior in auctions has long been studied using experimental methods (see Kagel and Levin [1] for a recent detailed survey). In this paper, we continue this tradition and study sequential auctions in which bidders fulfill their needs once they have obtained one good and drop out of the market for subsequent ones. We may refer to this type of auctions as auctions with *capacity constraints* since obtaining a good saturates the capacity of the participant for a time period, restricting the possibility to bid in an auction for a while, that is, until at least part of the capacity is freed. In their recent survey, Kwasnica and Sherstyul [2] call them "multi-unit auctions with single-unit demand".

Several real-life cases might be mentioned concerning capacity restrictions of this sort. Construction companies submit bids for building a major infrastructure. They know about their current cost and potential benefit of being the awardee, but they also know that there will be more contracts in the future, and that once a contract is awarded their limited capacity and business size may prevent them from bidding in other, perhaps more lucrative ones.

Similarly, transportation companies may compete for special transportation jobs. Companies with a reduced fleet must trade-off the benefit of obtaining the contract and the opportunity cost of bidding in future ones. In this market, companies like uShip.com provide the platform in which transportation tasks are auctioned.

Radio taxi companies face a related allocation situation when a new service comes along. The receiving center puts a call on the radios of the vehicles. A taxi driver may decide to accept the service or reject it, hoping that a closer or more rewarding service will show up soon.

Reverse auctions are also embedded in Customer Response Management (CRM) software like *salesforce*, where buyers of raw materials put a requirement in the web to their registered suppliers, who must post a quote. These companies supply goods to different clients, so committing production resources to this current winning bid might hamper the possibility of obtaining a better sale in the future.

Participants in this dynamic assignment of goods understand the inter-temporal tradeoff between current and future options. The singularity of the situation is that they have information about the current good but uncertainty about the future ones, not knowing which type of good will be auctioned next.

The aim of our paper is to study this trade-off in a controlled laboratory setting given different allocation mechanisms. In particular, we want to determine if subjects realize that patience may pay off.

Analyzing sequential auctions is not new. In Keser and Olson [3], 8 participants bid sequentially for 4 goods in first-price sealed-bid auctions. The authors observed diminishing prices as the auction went along, and classified subjects according to their degree of risk aversion. Salmon and Wilson [4] replicated the study using an English auction. Patience or “wait-and-see” strategies were also analyzed under the same experimental rules in Neugebauer and Pezanis-Christou [5]. Finally, Brosig and Reis [6] studied two consecutive first-price auctions with two bidders who can undertake only one project each. In this literature, however, valuations for all goods are known at the outset. Early bids have a strong signaling component, so the set of all Perfect Bayesian Equilibria is large and difficult to characterize.

To our knowledge, Leufkens et al. [7-8] are the only existing dynamic auction experiments where valuations for the second good are drawn *after* the allocation of the first good. The authors study a two-good setting with *synergies*, where the agent who obtains the first good has an increased valuation for the second good. As a result, there is an incentive to overbid for the first good and capitalize with the second. The authors find that subjects face an exposure problem and do not fully incorporate in their bid the extra value of winning.

Our paper shares the sequential revelation of valuations with Leufkens et al. [7-8], capturing the idea that bidders are uninformed of the value of future opportunities when bidding for the current contract. However, we focus on the

opposite inter-temporal trade-off: winning good t prevents subjects from bidding for good $t+1$, making the object at t less desirable. In other words, there is an option value of waiting – very much on the lines of the literature on investment under uncertainty [9] – which implies that, for a given valuation, bids should be lower and profits should be higher for goods auctioned early compared to goods auctioned late.

Formally, the experiment is set with groups of 4 subjects sequentially bidding for 3 goods, both under the rules of first- and second-price sealed bid auctions. Valuations are provided to the subjects “one-at-a-time”, and the winner of a good cannot bid for the remaining one(s). Theory predicts lower bids for earlier goods due to the standard “option value of waiting” argument and for first-price auctions due to the obvious effect of higher payments conditional on bids.

Not surprisingly in light of the existing literature, we find that subjects in our experiment deviate from the theoretical predictions, with an overall tendency to overbid. Subjects also exhibit a similar bidding behavior across goods and mechanisms. In our setting, this means that overbidding is stronger in first- than in second-price auctions ([10-11], for example). Overbidding is also stronger in earlier than in later goods.

The similarity in bidding across auctions suggests an imperfect understanding of fine distinctions across mechanisms. The similarity in bidding across goods can be also due to an imperfect understanding. However, it may be the result of a subtler *dynamic self-selection effect*. Indeed, subjects with a tendency to overbid are likely to win early goods (1 and 2) and exit the auction, leaving only those who play close to equilibrium or underbid competing for the last good (3). If this hypothesis is correct, we should then observe that subjects who typically win good 3 bid closer to equilibrium for all goods than subjects who typically win goods 1 and 2, and also make higher profits. Overall, when competing against irrational bidders, patience (that is, waiting until the overbidders get a good and leave the auction) pays off.¹

Interestingly, our results support the self-selection hypothesis. Indeed, for each type of auction we divide subjects into those who often win goods 1 and 2 (early winners) and those often win good 3 or none of them (late winners). An analysis of the bidding behavior suggests that early winners overbid substantially more than late winners not only for goods 1 and 2 (which explains why they win them so often) but also for good 3. We also show that the average efficiency is much larger for good 3 than for goods 1 and 2, mostly because of a change in the composition of the population: irrational bidders (who overbid and “steal” the good despite their not having the highest valuation) obtain goods 1 and 2 and leave the

¹ This selection effect is related but different from that in Casari, Ham and Kagel [12], where subjects have an initial endowment and those with financially poor decisions go bankrupt and are forced to leave the auction. Our selection is more directed. In each round, two bidders out of four win goods 1 and 2. They exit and cannot bid for good 3.

subjects who bid closest to equilibrium compete for good 3. In other words, even though early winners also overbid whenever they reach good 3, the efficiency is high simply because they rarely reach good 3. Finally, the same change in the composition of the population implies that patience pays off: late winners obtain substantially higher profits than early winners and gains are larger in the last good than in the first two, even though the theoretical prediction is the opposite.

Finally, it is interesting to notice that a substantial fraction of irrational overbidders remain in the auction after the allocation of the first good, as witnessed by the remarkably similar levels of high overbidding, low efficiency and low payoffs in goods 1 and 2. In other words, removing the 25% of subjects who win good 1 is not enough to significantly reduce the departures from equilibrium. Efficiency is significantly improved and overbidding significantly reduced only after 50% of subjects have left the auction.

The paper is organized as follows. In Section II, we develop the theory of dynamic bidding in first- and second-price auctions. In Section III, we describe the experimental design. In Section IV, we perform an aggregate analysis and study the bidding functions, efficiency and payoffs by good and mechanism. In Section V, we conduct a cluster analysis and test our self-selection hypothesis to explain the differences in bidding, efficiency and profits across goods. In Section VI, we provide some concluding comments. Proofs and instructions are relegated to the appendix.

Theory

Consider the following auction setting. There are $T + 1$ risk-neutral bidders and T periods. In period 1, each bidder i learns his valuation for the good to be auctioned in that period. We assume that valuations are independently drawn from the distribution $G(\cdot)$ with support $[\underline{v}, \bar{v}]$. Each bidder i simultaneously submits a sealed-bid $b_i^1 \in \mathbf{R}$ and the good is allocated according to the rules of mechanism $M \in \{\mathbf{F}, \mathbf{S}\}$, where \mathbf{F} is a first-price sealed bid auction with no reserve price (highest bidder wins and pays his bid) and \mathbf{S} is a second-price sealed-bid auction with no reserve price (highest bidder wins and pays the second highest bid). The winner of the auction obtains the good and exits the auction. The T losers move to period 2 where new valuations are drawn for those bidders (independently both across bidders and across periods) from the same distribution $G(\cdot)$ and a new auction takes place under the same mechanism (\mathbf{F} or \mathbf{S}) as the previous one. The process continues until period T , the last period, where valuations are drawn for the two remaining bidders who then bid for the last remaining good.

This dynamic auction has two characteristics that we want to emphasize. First, each bidder is interested in at most one good: once they have won a good, they exit the auction and the game ends for them. This occurs in practice when the opportunity cost of obtaining a second good is prohibitively high. Second, bidder i 's valuation for good $t+1$ is unknown in period t . In terms of the examples mentioned in the introduction, one can think of the owner of one truck, one taxi or one

construction crew who learns and bids for jobs as they appear. Once a job has been secured, he does not have the means to bid for a second one.

Propositions 1 and 2 characterize the symmetric Nash equilibrium of the game for bidder i ($\in \{1, \dots, T+1\}$) in period t ($\in \{1, \dots, T\}$), under the first-price (**F**) and the second-price (**S**) auction mechanisms, respectively (proofs can be found in Appendix A).

Proposition 1 In a first-price auction (F), the unique symmetric equilibrium bidding function of bidder i with valuation v_i in period t , $b_t^F(v_i)$, and his equilibrium utility, $u_{it}^F(v_i)$, are:

$$b_t^F(v_i) = v_i - \frac{\int_{\underline{v}}^{v_i} G(s)^{T+1-t} ds}{G(v_i)^{T+1-t}} - V_{t+1}^F \quad (1)$$

and

$$u_{it}^F(v_i) = \int_{\underline{v}}^{v_i} G(s)^{T+1-t} ds + V_{t+1}^F \quad (2)$$

where

$$V_{t+1}^F = \int_{\underline{v}}^{\bar{v}} G(v_i) dv_i - \int_{\underline{v}}^{\bar{v}} G(v_i)^{T+1-t} dv_i \quad (3)$$

Proposition 2 In a second-price auction (S), the unique symmetric equilibrium bidding function of bidder i in period t , $b_t^S(v_i)$, and his equilibrium utility, $u_{it}^S(v_i)$, are:

$$b_t^S(v_i) = v_i - V_{t+1}^S \quad \text{and} \quad u_{it}^S(v_i) = \int_{\underline{v}}^{v_i} G(s)^{T+1-t} ds + V_{t+1}^S \quad (4)$$

where

$$V_{t+1}^S = \int_{\underline{v}}^{\bar{v}} G(v_i) dv_i - \int_{\underline{v}}^{\bar{v}} G(v_i)^{T+1-t} dv_i \quad (5)$$

The results are extensions of the standard one-shot first-price and second-price sealed bid auctions with no reserve price. Indeed, in the first-price auction, we can interpret V_{t+1}^F as the value for a bidder at period t of *not winning* the current auction and, instead, moving to period $t+1$. In the standard auction, this is nil, which is why the value of not winning the auction in the last period (T), V_{T+1}^F , is zero. At every other period t ($< T$), V_{t+1}^F is positive and larger the greater the number of periods left ($V_t^F > V_{t+1}^F$). Notice also that V_{t+1}^F is constant, reflecting the fact that at period t bidder i does not know his future valuations.

Seen under this light, the equilibrium bid in the first-price auction at period t , $b_t^F(v_i)$, takes a familiar form. The first two terms correspond to the static equilibrium bid when the bidder faces $T+1-t$ rivals, and the last term reflects the opportunity cost of winning. In other words, bidder i shades his bid relative to his valuation for two reasons: first to optimize the standard trade-off between probability of winning and net gain conditional on winning, and second to reflect the positive value of moving to period $t+1$ and participating in a new auction. The equilibrium utility at period t , $u_{it}^F(v_i)$, is also familiar. It simply corresponds to the

standard expected utility of a bidder in a first-price auction against $T+1-t$ rivals to which we add V_{t+1}^F , the value of moving to the next period.²

The analysis of the second-price auction is analogous. If in a symmetric static equilibrium subjects bid their valuation, in our problem they decrease that bid by V_{t+1}^S , the option value of moving to $t+1$. As in the one-shot auctions, the expected payoff of bidders in each period is identical across mechanisms ($u_{it}^F(v_i) = u_{it}^S(v_i)$) and so is the expected value of moving to the next period ($V_{t+1}^F = V_{t+1}^S$). Notice also from Propositions 1 and 2 that the utility in period t can be rewritten as:

$$u_{it}^F(v_i) = u_{it}^S(v_i) = \int_{\underline{v}}^{\bar{v}} G(v_i) dv_i - \int_{v_i}^{\bar{v}} G(s)^{T+1-t} ds, \quad (6)$$

which immediately implies that $u_{it}^F(v_i) > u_{it+1}^F(v_i)$ for all t and $G(\cdot)$: as we move from one period to the next, the expected utility of a subject decreases because he faces fewer options to obtain one good.

The analysis in this section assumes no reserve price, which we know is suboptimal from the seller's viewpoint. Determining the equilibrium with optimal reserve price poses no extra difficulty. Remember, however, that the objective is not to find optimal mechanisms but to build a simple framework that we can export to the laboratory. We therefore opted for an environment with no reserve price.

Finally, suppose that a subject with valuation v_i wins the auction at date t . His utility in the first-price auction is then $v_i - b_t^F(v_i)$ (which, by the equivalence of the mechanisms, is also equal to his expected utility in the second-price auction). From an ex-ante perspective, $E[u_t^{win}]$, the expected payoff of the winner at t is:

$$E[u_t^{win}] = \frac{\int_{\underline{v}}^{\bar{v}} (v_i - b_t^F(v_i)) G(v_i)^{T+1-t} g(v_i) dv_i}{\int_{\underline{v}}^{\bar{v}} G(v_i)^{T+1-t} g(v_i) dv_i} \quad (7)$$

Notice that $E[u_t^{win}] > E[u_{t+1}^{win}]$. The expected payoff of the winner decreases over periods for two reasons. First, because bids increase with t and therefore the net gain of the winner decreases with t . Second, because as t increases, the number of bidders decreases. By the largest order-statistics, this means that the distribution of the highest valuation (which in equilibrium is the valuation of the winner) shifts towards lower values.

Before describing the experimental environment, we develop the simple numerical example that will be used in our experiment.

Example 1 Suppose that $\underline{v} = 30$, $\bar{v} = 90$ and $T = 3$, we get:

$$V_1^F = V_1^S = 18; V_2^F = V_2^S = 15; V_3^F = V_3^S = 10; V_4^F = V_4^S = 0.$$

$$b_1^F(v_i) = \frac{3}{4} v_i - \frac{15}{2}; b_2^F(v_i) = \frac{2}{3} v_i; b_3^F(v_i) = \frac{1}{2} v_i + 15.$$

² Indeed, since all bidders at period t decrease their bid by V_{t+1}^F compared to a static auction and there is no reserve price, the winner's payoff is increased by that amount and the losers' payoff is also increased by that amount (the value of going to the next stage).

$$b_1^S(v_i) = v_i - 15; b_2^S(v_i) = v_i - 10; b_3^S(v_i) = v_i.$$

$$u_1^F(v_i) = u_1^S(v_i) = \frac{1}{4} 60 + \frac{1}{4} \frac{(v_i-30)^4}{60^3}; u_2^F(v_i) = u_2^S(v_i) = \frac{1}{6} 60 + \frac{1}{3} \frac{(v_i-30)^3}{60^2};$$

$$u_3^F(v_i) = u_3^S(v_i) = \frac{1}{2} \frac{(v_i-30)^2}{60}.$$

$$E[u_1^{win}] = 27; E[u_2^{win}] = 25; E[u_3^{win}] = 20.$$

Experimental design and procedures

We conducted 4 sessions with 12 subjects and 2 sessions with 16 subjects for a total of 80 participants. All sessions were held at a computer lab in the Vicálvaro Campus of the Universidad Rey Juan Carlos (URJC) in Madrid (Spain). Subjects were recruited via email after posting a message on the university website calling for participation on a seminar about auctions but without any notification neither that the experiment was going to be held nor that they were going to get paid. Those who showed up were informed about the experiment and were given the option to withdraw. All decided to participate.

In each session, subjects had to bid on 8 rounds according to the rules of a first-price (**F**) or a second-price (**S**) auction. To control for order effects, in 3 sessions subjects started with 4 rounds of **F** followed by 4 rounds of **S** whereas in the other 3 sessions the order was reversed.

At the beginning of each round, subjects were randomly and anonymously matched into groups of 4. Subject knew about the procedures but did not know the identity of the subjects they were matched with.

For each group and each round, the auction consisted in the allocation of 3 goods to the 4 subjects in the group, with 3 subjects obtaining one good and the other subject obtaining none. The allocation mechanism followed closely the procedure described in Section II. When the first good was auctioned (from now on G1), all 4 subjects in the group received a random valuation for that good (and not for the other two to come). After sealed bidding, G1 was assigned to the highest bidder and the awarded price was the highest bid (**F**) or second highest bid (**S**), depending on the mechanism being used. The winner of the auction could not bid anymore in that round. Valuations were then drawn for the second good (from now on G2) for the remaining 3 subjects. Once again, G2 was awarded according to the same mechanism (**F** or **S**) and the winner was withdrawn for the round. Finally, for the third good (from now on G3), the last one, only two subjects remained. They both bid for the good and one got it while the other finished the round with no good.³

The valuations for each good in each round were denominated in tokens and randomly drawn from a uniform distribution in [30,90]. Bids were constrained to be

³ So, within one particular round, all three goods were allocated using the same mechanism (first- or second-price auction) but with different number of bidders (4, 3 or 2).

in the range $(0,150]$, rounded to 2 decimal places. For each good in each round, the payoff of the winner was value minus own bid (**F**) or value minus second highest bid (**S**). The profit of the loser(s) and the subject(s) not bidding (i.e., those who won a previous good in that round) was 0.

At the end of the session, the subjects were paid according to one round that was chosen at random by throwing three coins and showing the resulting sequence of heads and tails to the participants. The payoff was converted to euros at a ratio of 1 euro per 4 tokens. Subjects also earned a show-up fee of 2 euros.

The timing of the experiment was the following. First, the experimenter read the instructions (see Appendix B for a sample copy of the translation to English) showing sample screenshots in an overhead projector. Then, subjects took a short quiz on paper to make sure that each one had completely understood the instructions. After that, subjects went through one practice round that did not count for the final payoff and then they participated in the 8 paid rounds of the experiment. Finally, the subjects were paid in cash and in private their total earnings. Sessions averaged 75 minutes and subjects' earnings averaged 5.6 euros, which is admittedly low for the standards in economics experiments.⁴

For each round and group, 9 bids were submitted: 4 bids for G1, 3 for G2 and 2 for G3. Since there were 80 participants in the experiment, there were a total of 20 groups, each playing 8 rounds. The total number of bids was 640 for G1, 480 for G2 and 320 for G3, of which half corresponded to mechanism **F** and the other half to **S**. The experiment was programmed and conducted with the software z-Tree [13].

Ethics Statement

The experiment falls in the category of non-medical behavioral experiments in social sciences. It was run with IRB approval from USC (# UP-08-00052) and consent of the ethical committee of URJC.

Aggregate results

Summary statistics

We first provide indicators that summarize the main features of the experiment, looking at goods and mechanisms separately.

The first indicator relates to the efficiency of the auctions (Table 1), defined as the percentage of times that the good is allocated to the subject with the highest valuation. The efficiency is similar for G1 and G2 and increases substantially for G3. **S** auctions are generally more efficient, although it is surprising to see that the efficiency for G2 is lowest. The efficiency for **F** auctions is always increasing. Note that an increase in efficiency over auctions can be partly attributed to the decrease in the number of bidders (for instance, random allocation would predict efficiencies

⁴ The two reasons for the low payment were a low conversion rate and a significant overbidding.

of 25%, 33% and 50% in G1, G2 and G3). However, the 90% allocation efficiency of G3 is remarkably high. As we will see all along the paper, differences across goods are partly due to the endogenous “exit” (through winning) of subjects.

Table 1. Average efficiency and response times

Good	EFFICIENCY (percentage)			RESPONSE TIMES (seconds)		
	FIRST	SECOND	Total	FIRST	SECOND	Total
G1	62.5	71.3	66.9	19.8	21.3	20.6
G2	68.8	62.5	65.6	14.6	14.8	14.7
G3	87.5	93.8	90.6	14.8	15.0	14.9
Overall	72.9	75.8	74.4	17.0	17.7	17.4

The second indicator relates to the average time spent by the subjects to place their bids. They are slower at the beginning. We conjecture this is the case because the decision problem is more complex in the presence of a higher number of rivals and/or because subjects take relatively more time to choose the whole strategy (at the beginning of the round) than to implement their choices (as the round proceeds). Unfortunately, our data does not allow distinguishing between these two hypotheses.

The third indicator shows the payoffs obtained by the winner in each mechanism and for each good, defined as the average valuation minus bid (**F**) or valuation minus second highest bid (**S**) of the winner (Table 2). It also shows the theoretical prediction, as described in Example 1. Negative empirical values indicate paying a price over one’s own value when winning.⁵ Differences from theory occur for two main reasons. First, overbidding results in smaller gains conditional on winning. Second, it may also imply stealing the good, that is, winning when another subject has a higher valuation. This means a non-zero rather than zero payoff for that subject but an overall loss in surplus.

⁵ We did not ask subjects with negative payoffs to pay money back. Limited liability may have had an impact on the behavior of some subjects, although we conjecture that a small one.

Table 2. Average payoff of winner in tokens

		FIRST	SECOND	Total
G1	Theory	27.0	27.0	27.0
	Experiment	-0.9	10.7	4.9
	Std. Error	1.7	2.0	1.4
G2	Theory	25.0	25.0	25.0
	Experiment	-3.7	10.3	3.3
	Std. Error	2.1	1.7	1.4
G3	Theory	20.0	20.0	20.0
	Experiment	1.8	23.8	12.8
	Std. Error	1.3	1.8	1.4
Overall	Theory	24.0	24	24.0
	Experiment	-0.9	14.9	7.0
	Std. Error	1.0	1.1	0.8

There is a high dispersion in payoffs in both auctions. We also notice large payoffs differences between mechanisms with substantial profits in **S** (although well below equilibrium in G1 and G2) and profits close to zero in **F**, despite the identical theoretical predictions. A possible explanation is that if only the winner overbids, he will incur in losses in **F** but not in **S**, where the price is determined by the second highest bid. In other words, irrationality of a fraction of subjects is likely to be more costly in **F** than in **S**. Last and perhaps most interestingly, payoffs increase over goods (especially between G2 and G3). This is in sharp contrast with the theoretical predictions, where the increasing bids and decreasing number of participants over goods imply decreasing expected profits of the winner.⁶ As we develop below, the composition of subjects in G3 is different, which may be responsible for this effect.

Aggregate bidding functions

An analysis of interest consists in determining the empirical bidding behavior as a function of the valuation drawn by the subject.

For the rest of the analysis, we refer to “overbid” as the difference between the experimental bid and the (unique, symmetric) equilibrium bid predicted by theory (Nash). Positive values indicate overbidding and negative values underbidding. Bidding is studied by comparing theoretical predictions and experimental behavior at the aggregate level

Figure S1 describes bidding as a function of valuation. From left to right, the first graph includes the six theoretical bidding functions (BF) derived in Example 1 of Section II (goods G1, G2, G3 under mechanisms **F** and **S**). Note that the equilibrium bidding functions are linear in valuation because the distribution is uniform. The second graph shows the empirical bidding functions. The X-axis

⁶ As we discussed above, with fewer competitors the likelihood of winning and expected payoff of each bidder is higher. However, the expected valuation of the highest bidder is lower and so is his expected profit *conditional on being the winner*.

represents valuations. To smooth out the empirical functions, we group valuations in bins of 5 units, starting at the minimum of 30 and ending at the maximum of 90 (so 30 to 35, 35.01 to 40, etc.). The Y-axis shows the average bids that correspond to all the valuations included in that bin. The third graph depicts the regression-based experimental bidding functions. Since the theoretical bidding function is linear, we use the following OLS regression to estimate the best fit of the empirical bidding functions:

$$b_{gm}^i = \beta_0 + \beta_1 v_{gm}^i + \varepsilon_{gm}^i \quad (8)$$

where i denotes the individual, $g \in \{G1, G2, G3\}$ the good and $m \in \{F, S\}$ the allocation mechanism. This estimate can then be compared to the theoretical ones.

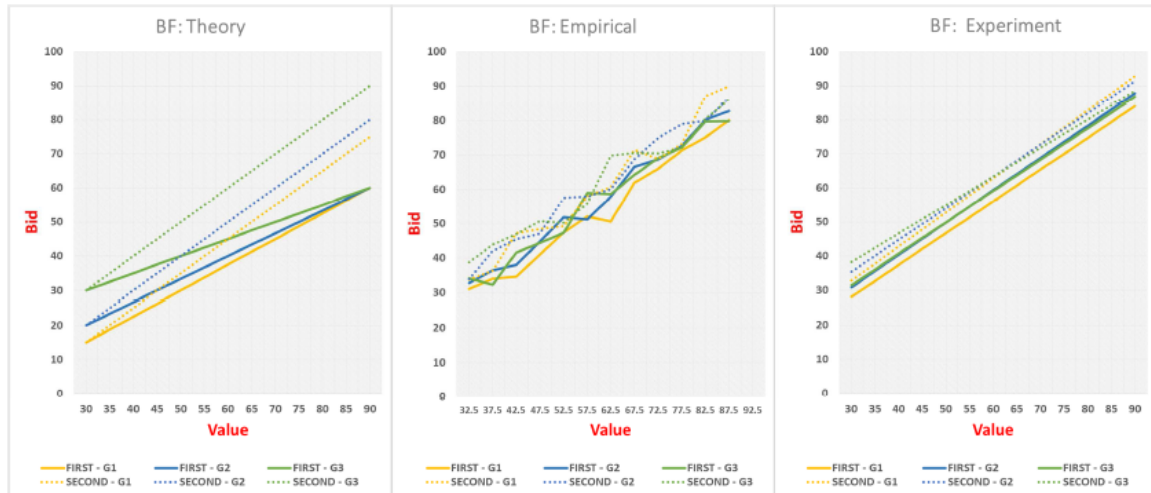


Figure S1. Bidding functions

Table 3 below includes the slopes and intercepts (at the minimum valuation of $v = 30$) of the theory and the OLS regression with their corresponding standard errors. It also includes the results of a t -test of comparison between the two.

Table 3. Comparison of bidding functions

		FIRST - G1	FIRST - G2	FIRST - G3	SECOND - G1	SECOND - G2	SECOND - G3
SLOPE	Theory	0.75	0.67	0.50	1.00	1.00	1.00
	Experiment	0.93*	0.95*	0.92*	1.00	0.93	0.84†
	Std. Error	0.04	0.03	0.04	0.06	0.04	0.05
INTERCEPT (at $v = 30$)	Theory	15.00	20.00	30.00	15.00	20.00	30.00
	Experiment	25.79*	28.53*	29.19	30.36*	33.15*	36.16*
	Std. Error	1.34	1.14	1.43	1.94	1.26	1.89

* significantly higher than theory at 5% level; † significantly lower than theory at 5% level.

Graphically, the theoretical bidding behavior shows differences between goods and mechanisms (lower bids in earlier goods and F auctions) whereas the aggregate experimental functions look similar across goods and mechanisms. This is consistent with the winner's payoff data: the higher relative overbidding in F and in G1-G2 results in lower profits in those treatments.

Our test of comparison between theory and data corroborates those findings. There is consistent overbidding as witnessed by the higher empirical intercept in 5 out of 6 cases. Subjects are also more responsive to changes in valuation than predicted by theory in **F**, where all the empirical slopes are above 0.9 and the theoretical are between 0.5 and 0.75. This is not the case in **S**, where slopes are not significantly different from theory in G1 and G2 and smaller than theory in G3. More generally, both slopes and intercepts are more similar across mechanisms and goods than predicted by theory.

Overbidding

In order to deepen our understanding of the differences between theoretical and empirical bidding behavior, we represent information using boxplots. Figure S2 shows a boxplot per mechanism and good, with the box height indicating the interquartile range ($q_3 - q_1$) and the line in the middle of the box illustrating the median (q_2). The whiskers' edges indicate maximums and minimums. For comparative purposes, the graph also includes the overall range ($\max - \min$) and the overall interquartile range ($\max q_3 - \min q_1$). Finally, the notches indicate the 95% confidence intervals on the median and are connected with straight lines to ease the comparison on central tendencies across goods and types of auctions.

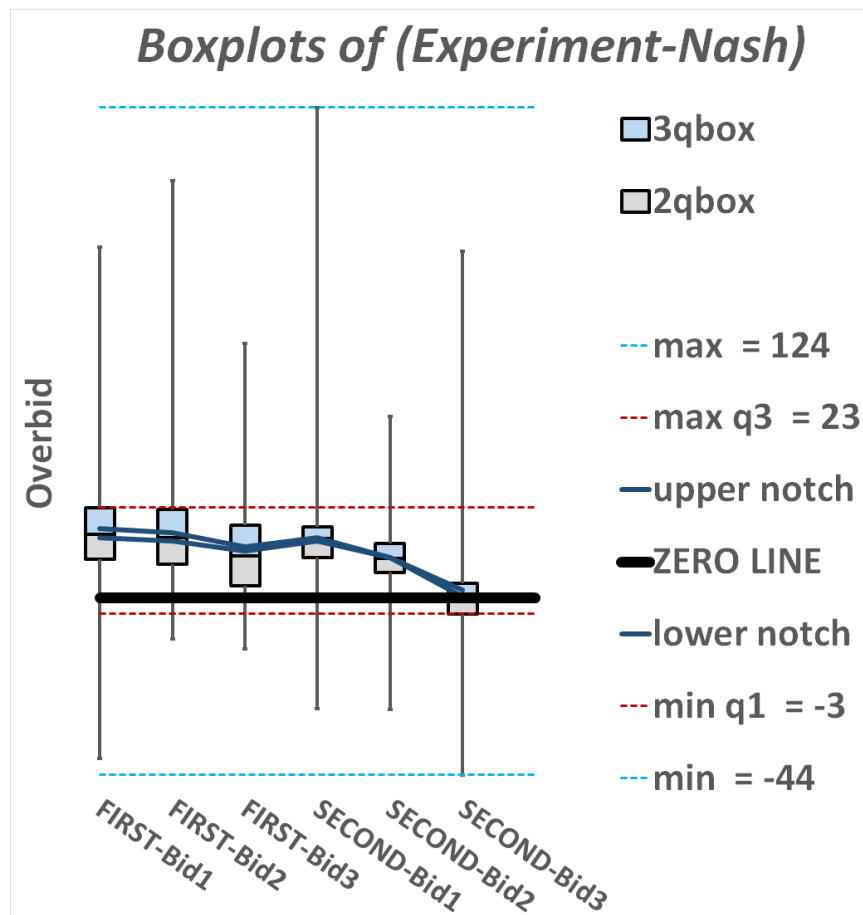


Figure S2. Boxplots of average overbids

From the boxplot, we notice an overbidding tendency that diminishes over goods. This is in line with the results in the previous subsection, where we showed that the empirical bidding strategies are similar across goods whereas the theoretical strategies are increasing across goods. Interestingly, the median bid for G3 in **S** is right on the equilibrium. Table 4 with the statistical comparison of medians confirms these findings: there is decreased overbidding across goods both in **F** ($1 = 2 > 3$ as shown in row 1) and in **S** ($4 > 5 > 6$ as shown in row 2). The tendency to overbid is also greater in **F** than in **S** for G2 and G3 (row 3).

Table 4. Comparison of medians of the bidding functions

Across goods (FIRST)	1 = 2	1 > 3	2 > 3
Across goods (SECOND)	4 > 5	4 > 6	5 > 6
Across mechanisms (G1, G2, G3)	1 = 4	2 > 5	3 > 6

This analysis reinforces the result of consistent overbidding for all valuations, mechanisms and goods, except for G3 in **S**. More interestingly, within a good, overbidding increases significantly with valuation in **F** (reaching a median overbid of 28 units for the highest valuation bin) but it is remarkably constant in **S** (Figure S3). Again, the result is consistent with the findings in the section that includes the aggregate bidding functions, which emphasized that bids are excessively sensitive to valuation in **F** (above 0.90 when the theory predicts 0.50 to 0.75). By contrast, in **S** a one-unit increase in valuation translates into an almost one-unit increase in bid, just like the theory predicts. For the case of G3 in **S**, the median bidding is remarkably close to Nash in all valuation bins.

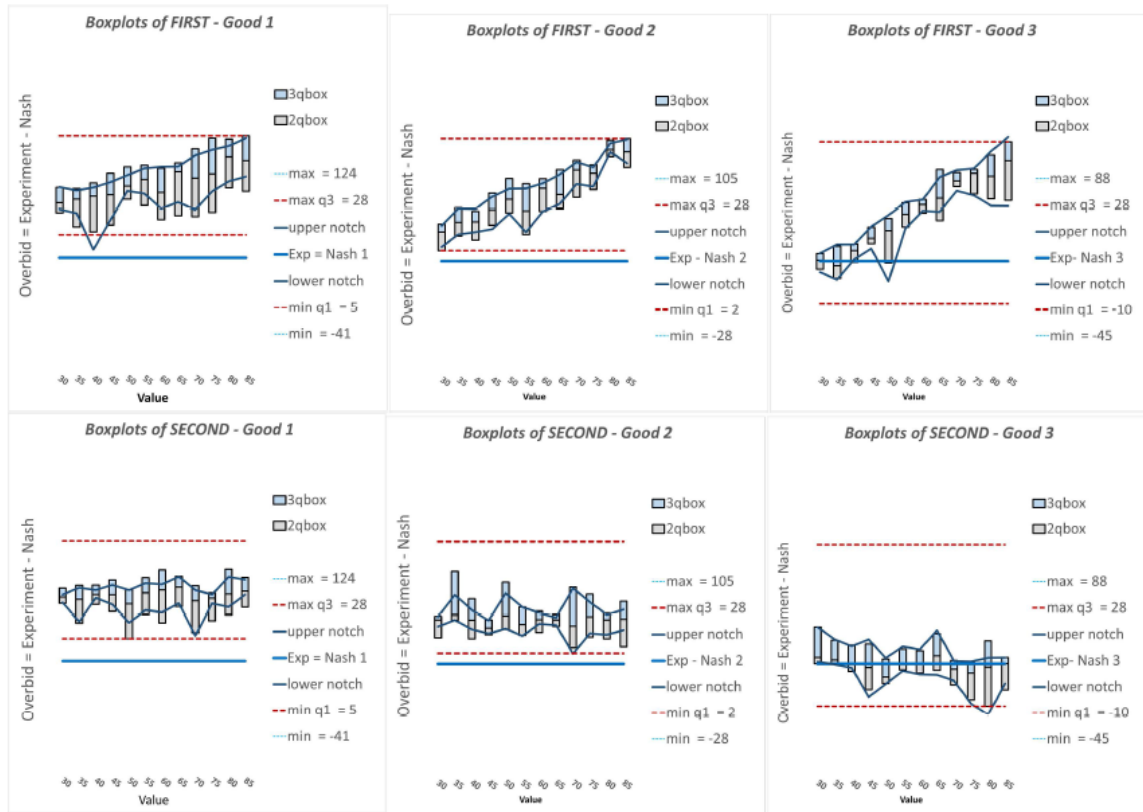


Figure S3. Boxplots of overbids per good and mechanism

Overall, the aggregate analysis suggests that bids are sensitive to valuations (excessively in the case of **F**) but, contrary to the theory, they are insensitive to differences in mechanisms (**F** vs. **S**) and goods (G1 vs. G2 vs. G3). Behavior is highly heterogeneous across subjects, with large dispersion in bids and significant overbidding in 5 out of 6 cases. The similarities in bids across mechanisms are roughly in line with the existing research in experimental auctions. Heterogeneity, however, raises an interesting selection problem in our setting: since winners of G1 and G2 do not participate in G3, the differences in departures from theory observed across goods may be due to differences in the *composition* of the bidding population. We will study this novel question in detail in the upcoming sections, but first we want to analyze the empirical payoffs obtained by our subjects.

Payoffs

The methodology to compare theoretical and experimental payoff functions (PF) is the same as the one followed to analyze bidding functions. We are interested in finding the payoff obtained by the winner of the auction, who in theory is also the highest valuation subject but in practice may not. The first graph of Figure S4 represents the theoretical net payoff of the winner (value minus own bid for **F** and value minus expected second-highest bid for **S** which, as we know from the theory section, are identical). The graph is therefore an affine transformation of the theoretical bidding function in **F** shown previously ($v_i - b_t^F(v_i)$ instead of only $b_t^F(v_i)$), so it is also linear in valuation. The second graph shows the empirical

payoffs. In the X-axis, valuations are grouped in bins of 5 once again⁷. In the Y-axis is the average net payoff of winners with valuations in that bin. The third graph includes a linear OLS regression to estimate the best fit of the winner's payoff function:

$$\pi_{gm}^i = \alpha_0 + \alpha_1 v_{gm}^i + \eta_{gm}^i \quad (9)$$

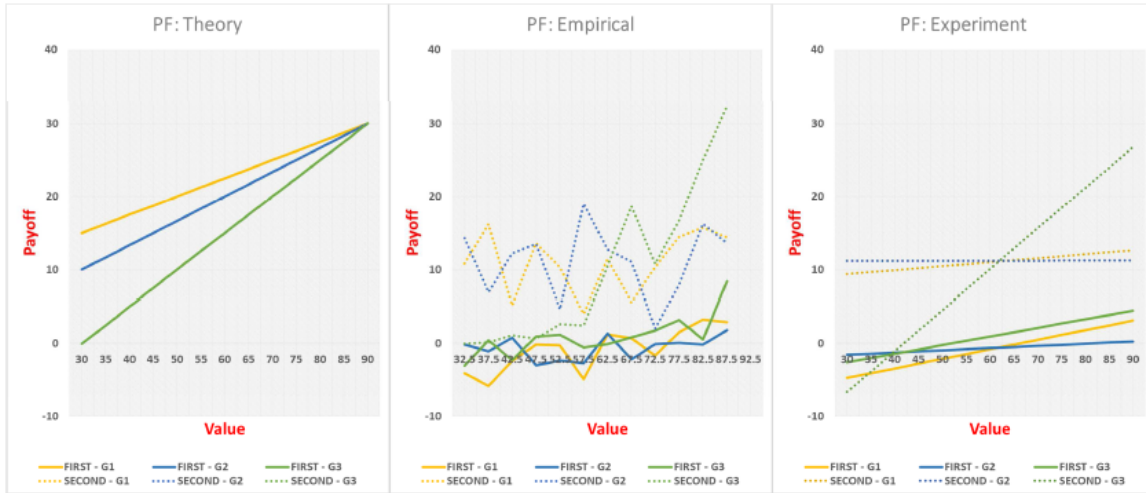


Figure S4. Payoff functions

It is important to notice the censored data aspect of the information reported compared to the bidding data presented previously, since we now only consider the values and bids of the highest bidder.

As before, we also present a table with slopes and intercepts (at $v = 30$) of the theory and the OLS regression with their corresponding standard errors (Table 5). It also shows the results of a t -test of comparison between the two.

Table 5. Comparison of payoff functions

		FIRST - G1	FIRST - G2	FIRST - G3	SECOND - G1	SECOND - G2	SECOND - G3
SLOPE	Theory	0.25	0.33	0.50	0.25	0.33	0.50
	Experiment	0.13 [†]	0.03 [†]	0.12 [†]	0.05 [†]	0.00 [†]	0.56
	Std. Error	0.03	0.03	0.03	0.07	0.09	0.08
INTERCEPT (at $v = 30$)	Theory	15.00	10.00	0.00	15.00	10.00	0.00
	Experiment	-4.68 [†]	-1.51 [†]	-2.56	9.40 [†]	11.15	-6.58 [†]
	Std. Error	1.11	0.91	1.18	2.47	2.98	2.68

[†]significantly lower than theory at 5% level

⁷ Since the functions are not too smooth, we also performed the same analysis with bins of 10. Few of these payoffs will correspond to low valuations, since for those valuations subjects are highly unlikely to win the auction. As anticipated, the payoff functions with bins of 10 were smoother due to the higher number of observations per bin but the OLS regressions and the comparisons theory vs. data were similar (results omitted for brevity but available upon request). We keep the results with bins of 5 to be coherent with the analysis performed in the previous sections.

Theory and data are vastly different in 5 out of 6 cases. In **F**, the payoffs of the winner are not far from zero and largely unresponsive to valuation, especially for G2. This is consistent with overbidding being high and increasing in valuation. Even subjects with valuations close to \bar{v} make very small profits. Differences in payoff functions across goods are also not significant. The picture is somewhat different for G1 and G2 in **S**. Payoffs are higher, though still below equilibrium predictions and similar for all valuations rather than increasing. Finally and in contrast with all the other cases, the winner's payoffs for G3 in **S** is remarkably close to theory, with empirical intercept and slope close to 0 and 0.5 respectively.

Cluster Analysis

Framework and basic statistics

The aggregate analysis shows overbidding, with a diminishing trend across goods. Bidding reaches behavior close to theory only for G3 in **S**. There are (at least) two possible explanations for this trend:

- Hypothesis 1: Bounded rationality. Subjects do not realize subtle differences between mechanisms and goods, and choose the same bidding strategy during the entire experiment.
- Hypothesis 2: Self-selection. Some subjects deviate more from equilibrium behavior than others. Since deviations take mostly the form of overbidding, these subjects win goods early in the round, leaving those who play closer to theory to bid for G3.

While Hypothesis 1 is standard in experimental auction models, Hypothesis 2 is more novel. There are indications of both effects in the aggregate analysis. On the one hand, the overall bidding behavior is similar between **F** and **S** for all goods. This suggests that subjects have difficulties differentiating between the two mechanisms, consistent with Hypothesis 1. On the other hand, behavior is closer to theory and efficiency is significantly higher in G3 than in G1 and G2. This suggests a relatively more rational and homogeneous behavior for the last good, consistent with Hypothesis 2.

If the hypothesis of self-selection holds true, we should observe that subjects who often obtain G1 or G2 overbid more than those who obtain G3 or do not obtain any good. This will partly happen by construction in G1 and G2. More interestingly, it should also occur in G3.

To study this issue we cluster subjects by the proportion of goods they obtain "early" vs. "late or never" in the round, and analyze their behavior to determine if their bidding strategies are different. More precisely, we define two attributes for each subject: "F3+0" is the percentage of rounds in which the subject has participated in G3 under **F** (either obtaining it -3- or not -0-) and "S3+0" is the percentage of times in which he has participated in G3 under **S**. Because our subjects play each mechanism 4 times, the percentages are 0, 25, 50, 75 or 100.

We then cluster the subjects based on these two dimensions using K-means, a clustering method that partitions the observations (here subjects) in K clusters and in which each observation belongs to the cluster with the nearest mean. According to the self-selection hypothesis, we expect two main groups. Subjects who often win G1 or G2 both in **F** and **S**, which we call EARLY-EARLY or EE. Subjects who often win G3 or do not win any good both in **F** and **S**, which we call LATE-LATE or LL. There might be also some unique bidding behaviors, with aggressive bidding and therefore high chances of early winning only in **F** (EARLY-LATE or EL) or only in **S** (LATE-EARLY or LE). A 4-means cluster seems therefore a reasonable option.⁸

Figure S5 depicts the two-dimensional distribution of subjects, grouped in 4 clusters (the size of each circle and the number inside represent the number of subjects). Notice that our method imposes the number of clusters but not the way in which subjects should be grouped with each other. So, subjects need not be clustered necessarily according to the EE, EL, LE, LL categories described above. It turns out, however, that this is how the model naturally classifies our subjects.

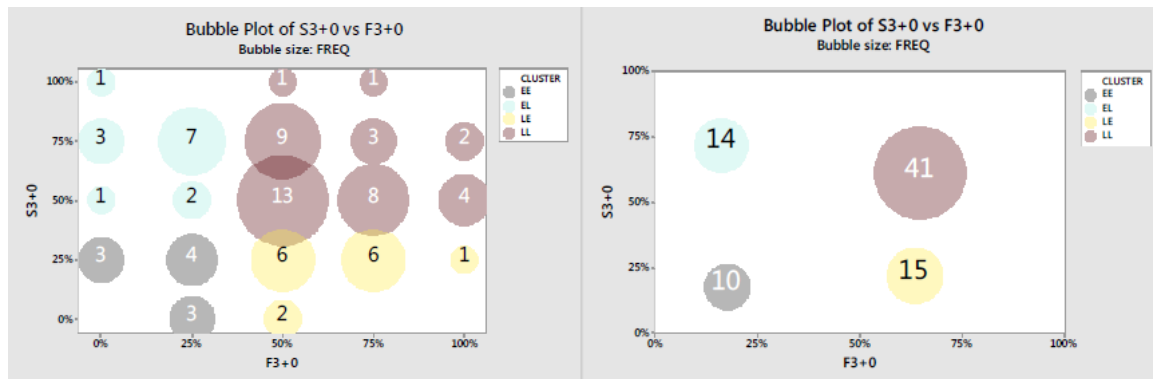


Figure S5. Clusters

The first thing to notice is that our clustering model puts in the L category the subjects who reach G3 exactly 50% of the time. Then, of the 80 participants, 41 subjects (51%) are LL, 10 subjects (13%) are EE, 15 subjects (19%) are LE, and 14 subjects (17%) are EL.⁹ Table 6 summarizes the percentage of goods obtained by subjects in each cluster and their average number of bids. Notice in particular that subjects who obtain the good late or never (L) bid on average 38% more often than those who obtain it early (E).

⁸ We tried also to cluster subjects in 2 or 3 groups, but the results were not as sharp and they did not lend to a clean test of our hypotheses.

⁹ This is not very different from what would occur if all subjects played the equilibrium strategies. Indeed, given our (endogenous) definition of clusters, we would statistically obtain 47% of LL, 10% of EE, 21% of EL and 21% of LE.

Table 6. Percentage of goods obtained per cluster and summary statistics

	EARLY - EARLY		EARLY - LATE		LATE - EARLY		LATE - LATE		Total
	FIRST (E)	SECOND (E)	FIRST (E)	SECOND (L)	FIRST (L)	SECOND (E)	FIRST (L)	SECOND (L)	
Goods 1+2	82.5%	82.5%	83.9%	28.6%	36.7%	78.3%	35.4%	39.0%	50.0%
Goods 3+0	17.5%	17.5%	16.1%	71.4%	63.3%	21.7%	64.6%	61.0%	50.0%
Avg. # bids	7.1		7.2		6.9		10.5		9.7
# Subjects	10		14		15		41		80

Overbidding

Given this grouping method, the first step in the analysis is to study bidding behavior across clusters. If all subjects in the experiment follow similar strategies (equilibrium or otherwise) then subjects who win early goods often are simply those who happened to get the more favorable draws of valuation. By contrast, if self-selection and bid heterogeneity is present, early (late) winners are subjects who bid more (less) aggressively. Table 7 presents the average overbidding in each good and mechanism separated by cluster.

Table 7. Overbid per cluster

Good	Overbid (Empirical - Nash)										
	EARLY - EARLY		EARLY - LATE		LATE - EARLY		LATE - LATE		Total		
	FIRST (E)	SECOND (E)	FIRST (E)	SECOND (L)	FIRST (L)	SECOND (E)	FIRST (L)	SECOND (L)	FIRST	SECOND	ALL
1	29.8	26.6	15.7	6.9	13.2	19.0	14.6	14.1	17.2	14.5	15.9
2	31.9	15.7	20.7	5.5	10.9	14.6	16.0	10.9	18.0	11.2	14.6
3	24.4	14.8	18.9	-3.8	9.7	5.4	11.0	1.2	12.6	0.6	6.3
Avg.	30.0	21.7	17.6	3.5	11.5	15.9	14.2	9.7	16.5	10.3	13.1

The average overbidding of an early winner ranges from 15.9 to 30.0 whereas that of a late winner is between 3.5 and 14.2. Overall and consistent with Hypothesis 2, overbidding is more than twice as high for EE than for LL both in **F** and in **S**. It is also striking that EL subjects are closest to theory in **S** but depart substantially in **F**.

Generally, overbidding decreases across goods, which again is consistent with Hypothesis 2 regarding the change in the composition of the population across goods. Interestingly, the decrease in overbidding is most pronounced between the second and third good, suggesting that the self-selection effect is still present after removing the 25% of subjects who substantially overbid and win the first good. Finally, notice that the difference in overbidding between EE and LL is significant not just on average but also on a good by good basis. In particular, an EE who reaches G3 will overbid about 13 tokens more than an LL who reaches G3. This

means that overbidding is truly a characteristic of the individual and not an artifact of our classification.¹⁰ Similar tendencies are observed with EL and LE.

Efficiency

The substantial overbidding of a large fraction of the population for G1 and G2 (the “early” subjects) may hamper the efficiency of the mechanisms. Table 8 shows the allocation efficiency in **F** and **S**. It represents for each cluster whether the bidder with the highest valuation effectively won the auction (Same) or not (Other), in which case it also states the cluster type of the “stealer”. Columns are added to summarize efficiency across goods.

Table 8. Efficiency analysis

WINNER IN THEORY	WINNER IN EXPERIMENT													
	FIRST	Same (eff.)	Other				% Eff.	SECOND	Same (eff.)	Other				% Eff.
			LL	LE	EL	EE				LL	LE	EL	EE	
LL	GOOD 1 -- 63%	18	4	3	6	8	46%	GOOD 1 -- 71%	31	4	2	1	8	67%
LE		9	3	1	3	1	53%		16	0	2	0	0	89%
EL		16	1	0	0	0	94%		4	3	1	0	0	50%
EE		7	0	0	0	0	100%		6	1	0	0	1	75%
LL	GOOD 2 -- 69%	24	4	2	6	3	62%	GOOD 2 -- 63%	15	5	5	2	7	44%
LE		7	2	0	1	3	54%		17	1	0	1	0	89%
EL		14	1	0	1	1	82%		8	3	4	0	1	50%
EE		10	1	0	0	0	91%		10	1	0	0	0	91%
LL	GOOD 3 -- 88%	40	3	3	1	1	83%	GOOD 3 -- 88%	47	1	1	1	3	89%
LE		21	1	1	0	0	91%		8	0	0	0	0	100%
EL		4	0	0	0	0	100%		16	2	0	0	2	80%
EE		5	0	0	0	0	100%		4	0	0	0	0	100%
Overall		73%							76%					

For G1 and G2 in **F**, efficiency is very high, between 82% and 100%, whenever an “early” subject (EE or EL) has the highest valuation. These subjects also frequently steal the goods that a “late” subject (LE or LL) should obtain. This decreases dramatically the efficiency in those cases to levels in the 46% to 62% range. Once we reach G3, most early subjects have already obtained a good. The presence of overbidders is not as widespread as in G1 and G2 so their competition is not as fierce, resulting in extremely high efficiency levels (83% to 100%).

The analysis is similar in **S**: high efficiency in G1 and G2 (75% to 91%) when an early subject (EE or LE) has the highest valuation and much lower efficiency (44% to 67%) when a late subject (EL or LL) has the highest valuation. Again, efficiency is uniformly high in G3 (80% to 100%).

¹⁰ Indeed, if each subject sometimes overbids and sometimes not, whenever a subject overbids he will be more likely to win G1 or G2 and therefore be classified as an E. However, if that were the case, there would be no reason why that subject would also overbid consistently and significantly more than the other subjects also in G3.

Overall, the efficiency analysis also supports the self-selection hypothesis. Despite the fact that E subjects exhibit the same amount of overbidding in G3 than in G1 and G2, the efficiency in G3 is dramatically increased. This can only be explained by the fact that most E subjects have already obtained a good and therefore do not participate in the auction of G3.

Payoffs

The last step of our analysis consists in studying the winner's payoffs in each cluster. We follow the same methodology as we did for overbidding and present a table of average payoffs across goods and mechanisms, separately for each cluster (Table 9).

Table 9. Average payoff of winner per cluster in tokens

Good	EARLY-EARLY		EARLY-LATE		LATE-EARLY		LATE-LATE		Total			Theory
	First (E)	Second (E)	First (E)	Second (L)	First (L)	Second (E)	First (L)	Second (L)	First	Second	All	
1	-13.8	-0.7	4.2	16.8	0.5	12.4	1.4	13.5	-0.9	10.7	4.9	27.0
2	-14.1	9.6	0.5	18.7	5.2	9.6	-3.6	7.8	-3.7	10.3	3.3	25.0
3	-11.2	26.3	-9.2	26.3	4.3	20.2	3.4	23.3	1.8	23.8	12.8	20.0
Avg.	-13.5	7.2	1.3	22.4	3.4	12.4	0.7	16.5	-0.9	14.9	7.0	24.0

The results in **F** are stark. Early winners incur in either severe losses (EE) or small gains (EL). They seldom reach G3 but when they do, they still overbid and lose substantial money. Late winners (LE and LL) avoid losses but they still do not obtain high payoffs for two reasons: because they still (moderately) overbid and because their rivals overbid. The difference between clusters is most significant in G3 where EE and LE lose 10 tokens on average whereas EL and LL, who face early winners less frequently, win 4 tokens. Overall, for the most rational bidders, patience pays off: contrary to the theory, earnings in G3 are higher than in G1 or G2 due to the self-selection effect.

Results are slightly more mixed in **S**. On the one hand, the tendency of late winners (EL and LL) to overbid less than early winners (EE and LE) implies that the former obtain larger gains than the latter: 22.4 and 16.5 vs. 7.2 and 12.4. On the other hand and contrary to **F**, patience pays off for all subjects: in all four clusters, the highest profits are obtained in G3 under **S** (and these gains are typically higher than predicted by theory). This counterintuitive result has a simple explanation. In G3 there are only two bidders, so if an EE or LE wins the good over an EL or LL, his bid is irrelevant for payoff purposes since he will pay the price set by his rival's bid. In other words, in G3 mistakes of the form of overbidding are less costly under **S** than under **F**, and this is reflected in the gains of "early" subjects in G3.

Summary

The cluster analysis of this section suggests that two effects contribute to the insufficient distinction in bidding behavior across goods and mechanisms. First and

as extensively documented in previous experiments on auctions [1], there is evidence of bounded rationality. Subjects have problems realizing the distinction between first and second price auction, and the corresponding differences in optimal bidding behavior. We show that this problem is most severe in G1 and G2 but persists in G3, despite the different composition of the subject population. Second and more interestingly, we emphasize a selection effect across goods. Subjects who obtain the good early tend to severely overbid in all goods, including the last one. They also steal goods, lowering the allocation efficiency and decreasing the aggregate payoffs when their type is prevalent (first and second good) but less so when their type is less common (third good).¹¹

Conclusion

In this paper we have provided a simple framework to study sequential auctions with capacity constraints. We have identified a novel self-selection effect, whereby irrational overbidders obtain early good(s) and exit the auction, leaving the most rational ones (and maybe also the underbidders) competing for the late good(s). In this setting, the patience of rational bidders pays off: both the efficiency and the payoff of the winner are higher for the last good than for the first two. The setting may not be appropriate to capture markets with highly experienced bidders or markets with a continuous flow of entry and exit of bidders. By contrast, it is highly suitable for markets with a fixed number of capacity constrained, moderately experienced players.

The self-selection result raises a number of theoretical and experimental questions. First, it would be interesting to develop a behavioral theory of boundedly rational bidding in markets with capacity constraints. Indeed, mistakes are not equally costly if they take the form of under- or over-bidding. Their relative cost and the efficiency consequences are also different in first- and second-price auctions. As a result, the best response of rational bidders to this boundedly rational behavior will also depend on the composition of over- and under-bidders in the population as well as their likelihood to exit early. Second, it would be instructive to extend the framework to the case where goods are drawn from different distributions. It is a priori unclear if the departures from equilibrium behavior will be exacerbated if the “best” goods (those drawn from $H(v_i)$, with $H(v_i) < G(v_i)$ for all v_i) are offered early and the “worst” goods are offered late or vice versa. It also raises interesting questions for the auctioneer. Should a profit-maximizing seller start or end with the best goods? How does this depend on the fraction of overbidders in the population? Finally, our experiment has only 8 rounds leaving little room to our subjects for learning to bid optimally. It would be interesting to determine if in an experiment with more rounds (say, 20 or 30) overbidders learn the costs of their behavior and converge over time to the equilibrium strategy. This and other related questions will be the object of future investigation.

¹¹ Note, however, that the self-selection of subjects is not uniform across mechanisms. Indeed, 36% of our subjects are “stealers” in one mechanism but not in the other (LE and EL).

Acknowledgements

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References

1. Kagel JH, Levin D (2011) Auctions: A Survey of Experimental Research, 1995 - 2010, to appear in *The Handbook of Experimental Economics, vol 2*.
2. Kwasnica T, Sherstyul K (2013) Multiunit auctions. *Journal of Economic Surveys* 27 (3): 461-490
3. Keser C, Olson M (1996) Experimental examination of the declining-price anomaly. *Contributions to Economic Analysis* 237: 151-176
4. Salmon TC, Wilson BJ (2008) Second chance offers versus sequential auctions: theory and behavior. *Economic Theory* 34(1): 47-67
5. Neugebauer T, Pezanis-Christou P (2007) Bidding behavior at sequential first-price auctions with(out) supply uncertainty: A laboratory analysis. *Journal of Economic Behaviour and Organization* 63 (1): 55-72
6. Brosig J, Reis PJ (2007) Entry decisions and bidding behavior in sequential first-price procurement auctions: an experimental study. *Games and Economic Behaviour* 58(1): 50-74
7. Leufkens K, Peeters R (2007) Synergies are a reason to prefer first-price auctions. *Economics Letters* 97(1): 64-69
8. Leufkens K, Peeters R, Vorsatz M (2012) An experimental comparison of sequential first- and -price auctions with synergies. *B E Journal of Theoretical Economics* 12(1): Article 2
9. Dixit AK, Pindyck RS (1994) *Investment under Uncertainty*. Princeton.
10. Güth W, Ivanova-Stenzel R, Wolfstetter E (2005) Bidding behavior in asymmetric auctions: An experimental study. *European Economic Review* 49(7): 1-27
11. Chernomaz K (2006) On the effects of joint bidding in independent private value auctions: An experimental study. Unpublished manuscript. Ohio State University.
12. Casari M, Ham JC, Kagel JH (2007) Selection bias, demographic effects, and ability effects in common value auction experiments. *American Economic Review* 97(4): 1278-1304
13. Fischbacher U (2007) z-Tree: Zurich Toolbox for Ready-made Economic Experiments. *Experimental Economics* 10(2): 171-178.

Appendix A: Proofs of Propositions 1 and 2

First-price auction. Consider period t ($\leq T$). Consider agent i with $T - t + 1$ opponents. Valuations are independently drawn from distribution $G(\cdot)$ in $[\underline{v}, \bar{v}]$. Agent i anticipates that agent $j \neq i$ bids $b_t(v_j)$ where $b_t(\cdot)$ is a monotonic increasing function (the same for all j).

If i announces b_i and gets the good, his surplus is $v_i - b_i$. If he does not get it, his surplus is V_{t+1} , the value of going to the next round ($t + 1$). The utility of i is then given by:

$$u_i^t(v_i, b_i) = (v_i - b_i)G(b_t^{-1}(b_i))^{T-t+1} + V_{t+1} \left[1 - G(b_t^{-1}(b_i))^{T-t+1} \right] \quad (10)$$

Bidder i chooses b_i such that $\frac{\partial}{\partial b_i} u_i^t(v_i, b_i) = 0$. Differentiating $u_i^t(v_i, b_i)$ with respect to v_i and using the previous optimality condition, we get:

$$\frac{du_i^t}{dv_i} = \frac{\partial u_i^t}{\partial v_i} = G(b_t^{-1}(b_i))^{T-t+1} \geq 0 \quad (11)$$

We assume that in each round one good is allocated with certainty (which is true if we impose no restrictions on bids). At equilibrium, \underline{v} never gets the good in that period. Therefore, the utility of a bidder with valuation \underline{v} is V_{t+1} . Overall,

$$u_i^t(v_i, b_t(v_i)) = \int_{\underline{v}}^{v_i} G(s)^{T-t+1} ds + V_{t+1} \quad (12)$$

Then, for all $v_i \geq \underline{v}$, the optimal bid is given by:

$$\begin{aligned} (v_i - b_t(v_i))G(v_i)^{T-t+1} + V_{t+1}[1 - G(v_i)^{T-t+1}] &= \int_{\underline{v}}^{v_i} G(s)^{T-t} ds + V_{t+1} \\ \Leftrightarrow b_t(v_i) &= v_i - \frac{\int_{\underline{v}}^{v_i} G(s)^{T-t+1} ds}{G(v_i)^{T-t+1}} - V_{t+1} \end{aligned} \quad (13)$$

which means in particular that $b_t(\underline{v}) = \underline{v} - V_{t+1}$. This equilibrium implies a positive bid ($b_t(\underline{v}) > 0$) as long as $\underline{v} > V_{t+1}$ (in the numerical application of the experiment, we make sure that this condition is satisfied so that the non-negative bid constraint is not binding). The equilibrium utility is:

$$u_i^t(v_i) = \int_{\underline{v}}^{v_i} G(s)^{T-t+1} ds + V_{t+1} \quad (14)$$

And the continuation value is:

$$V_t = V_{t+1} + \int_{\underline{v}}^{\bar{v}} \int_{\underline{v}}^{v_i} G(s)^{T-t+1} ds g(v_i) dv_i > V_{t+1} \quad (15)$$

Integrating by parts, we get:

$$V_t = V_{t+1} + \int_{\underline{v}}^{\bar{v}} G(v_i)^{T-t+1} dv_i - \int_{\underline{v}}^{\bar{v}} G(v_i)^{T-t+2} dv_i \quad (16)$$

which, using a recursive argument, implies:

$$V_t = V_{T+1} + \int_{\underline{v}}^{\bar{v}} G(v_i) dv_i - \int_{\underline{v}}^{\bar{v}} G(v_i)^{T-t+2} dv_i \quad (17)$$

Notice that, by definition, $V_{T+1} = 0$ since it represents the continuation payoff of not getting the good in period T (the last one). Therefore:

$$V_t = \int_{\underline{v}}^{\bar{v}} G(v_i) dv_i - \int_{\underline{v}}^{\bar{v}} G(v_i)^{T-t+2} dv_i \quad (18)$$

Finally, inserting the continuation value V_{t+1} into the utility $u_i^t(\cdot)$ we get:

$$u_i^t(v_i) = \int_{\underline{v}}^{\bar{v}} G(v_i) dv_i - \int_{v_i}^{\bar{v}} G(s)^{T-t+1} ds \quad (19)$$

Second-price auction. The proof for the second-price auction follows a similar line (it is straightforward to see that it is a weakly dominant strategy for subject i to bid his modified valuation $b_i^t = v_i - V_{t+1}$). Finally, notice that both auction formats yield the same expected utility to bidders and therefore also give the same expected revenue to the seller.

Appendix B: Instructions

You¹² are going to participate in an experiment in which you will have to make decisions in groups, and you will be paid in cash at the end of the experiment. Each participant may obtain different amounts due partly to its own decisions, partly due to the decisions of others, and partly due to the luck of the draws. The experiment is computer based and all the interactions among participants will be through the PC. It is important that you do not talk and that you do not try to communicate with other participants throughout the experiments.

We will start with a short period of instructions, in which you will be instructed on the rules of the experiment, and you will be taught on how to use the computers. It is very important that you pay close attention. If any questions arise, raise your hand and the answer will be given out loud for everyone. If any doubts strike your mind, raise your hand and I will help you with the computer.

At the end of the session, you will be paid according to just one of the 8 rounds that cover the experiment, chosen at random, plus an additional 2 euros as a participation reward. The payment will be performed on an individual basis and in private. You are not obliged to disclose whatever you have obtained.

The earnings during the experiment are measured in monetary units or tokens. According to your decisions you may win or lose tokens. At the end of the experiment, you will be paid in euros according to an exchange rate of 1 euro for every 4 tokens that you have earned during the round that has been selected at random.

The experiment will consist of 8 rounds of auctions. In each round, you will be randomly assigned to a group of 4 participants. You will not know the identity of the other 3 participants of your group. Since you are 12/16 participants, 3/4 groups will be formed in each round. Your reward depends exclusively on the decisions of the participants of your group and on the luck of the draws. Whatever happens on the other groups does not affect to your rewards, neither your behavior will affect the results on the other groups.

During each round, 3 goods will be auctioned among the 4 participants of each group sequentially, that is, one-at-a-time, and each participant will be allowed to buy just one of the three goods at stake. Therefore, at each round and for each

¹² What follows are the instructions in English. The Spanish version, the one that was used, is available upon request. The screenshots have not been translated.

group of 4 participants, 3 of the players will get one good and the fourth player will not get any. For each good at stake, only those participants that have not previously obtained a good may bid.

Let's now explain the rules of each round. At the beginning of each round, the PC will assign each participant to a group with other 3 participants, For each good being auctioned, the computer will assign a random valuation between 30 and 90 tokens. The interactive screen (Figure S6) will ask the participant to enter a bid, which must be positive and less than 150 tokens, and press the confirmation button.

NÚMERO DE PARTICIPANTE 2

INSTRUCCIONES:
CADA PARTICIPANTE SÓLO PUEDE COMPRAR UN BIEN
HAY 4 PARTICIPANTES
HAY 3 BIENES
LAS VALORACIONES SE ASIGNAN ALEATORIAMENTE ENTRE 30 Y 90.
LAS PUJAS DEBEN SER POSITIVAS.

SUBASTA DE PRECIO MÁXIMO:
GANADOR: PARTICIPANTE CON LA MAYOR PUJA; LOS EMPATES SE RESUELVEN DE FORMA ALEATORIA
PRECIO DE COMPRA: LA CANTIDAD DE LA PUJA MÁS ALTA

Ronda 1 de 2 Tiempo Restante 971

BIEN 1 de 3
Tu valoración, en u.m., es 65.03
Introduce una puja en u.m.
Confirmar

Ronda	Subasta	Beneficio
1	PUJAMÁS ALTA	0.00

Figure S6. Placing a bid

We are facing a sealed-bid auction, since the bids are anonymous and secret, and there is a set period of time to submit the bid. That is why it is critical to remain in silence.

Before submitting the bid, it is convenient to read the information on the screen, to correctly identify the good being auctioned and to fully understand the rules:

- 4 participants in each group and 3 goods per round
- The valuations are random between 30 and 90 tokens.
- The auction type currently under way: First Price and Second Price. The peculiarities of each of the two types will be explained later.

The rest of the available information shown on the screen is:

- The round or period
- The remaining time to submit a bid
- The good that it is being auctioned out of the possible 3
- The history of the experiment, indicating the profit obtained per round.

If a participant has previously obtained a good, the screen is different (Figure S7), just indicating the history, since the subject is not allowed to bid nor obtain a second good.

NÚMERO DE PARTICIPANTE 4

INSTRUCCIONES:
 CADA PARTICIPANTE SÓLO PUEDE COMPRAR UN BIEN
 HAY 4 PARTICIPANTES
 HAY 3 BIENES
 LAS VALORACIONES SE ASIGNAN ALEATORIAMENTE ENTRE 30 Y 90.
 LAS PUJAS DEBEN SER POSITIVAS.

SUBASTA DE PRECIO MÁXIMO:
 GANADOR: PARTICIPANTE CON LA MAYOR PUJA. LOS EMPATES SE RESUELVEN DE FORMA ALEATORIA.
 PRECIO DE COMPRA: LA CANTIDAD DE LA PUJA MÁS ALTA.

Ronda 1 de 2 Tiempo Restante 000

BIEN 2 de 3

No puedes pujar por este bien porque ya has obtenido uno

Tu beneficio acumulado es: 3.89

Ronda	Subasta	Beneficio
1	PUJAS MÁS ALTA	3.89

Figure S7. Not allowed to bid as a previous winner

When the participants confirm their bids or their profits, a wait screen will show up (Figure S8), screen that will disappear whenever each and every participant press the corresponding confirmation button.

Por favor, espere

Ronda	Subasta	Beneficio
1	PUJAS MÁS ALTA	9.99

Figure S8. Wait screen

When all the bids have been submitted, the computer will assign the good to the buyer or winner, whoever placed the highest bid. The price to pay in tokens will depend on the type of auction mechanism of the current round:

- Under First Price or Maximum Price, the Price coincides with the bid placed by the winner.
- Under Second Price or Vickrey, the Price corresponds not to one's own bid but to the second highest bid.

The profit obtained by the winner or buyer is equal then to the value minus the Price, being positive if the Price is lower than the valuation and negative if the price is above the valuation.

Following you will see simple screenshots with examples that may show up after the assignment of the good to the winner, screens that vary depending on the auction mechanism and the participant is the winner or not.

The first screenshot (Figure S9) will be seen by the winner of a First Price auction, and the profit is one's own valuation minus one's own bid.

NÚMERO DE PARTICIPANTE 3

INSTRUCCIONES:
CADA PARTICIPANTE SÓLO PUEDE COMPRAR UN BIEN
HAY 4 PARTICIPANTES
HAY 3 BIENES
LAS VALORACIONES SE ASIGNAN ALEATORIAMENTE ENTRE 250 Y 350.
LAS PUJAS DEBEN SER POSITIVAS.

SUBASTA DE PRECIO MÁXIMO:
GANADOR: PARTICIPANTE CON LA MAYOR PUJA; LOS EMPATES SE RESUELVEN DE FORMA ALEATORIA.
PRECIO DE COMPRA: LA CANTIDAD DE LA PUJA MÁS ALTA

Ronda 1 de 2 Tiempo Restante 018

BIEN 1 de 3
Tu valoración fue 323.35
Tu puja fue 307.24
El precio de compra ha sido 307.24

Tú eres el COMPRADOR
Tu beneficio es de 16.11

Ronda	Subasta	Beneficio
1	PUJAS MÁS ALTA	16.11

Figure S9. Winner of F

The second (Figure S10) corresponds to a non-winner in a First Price auction.

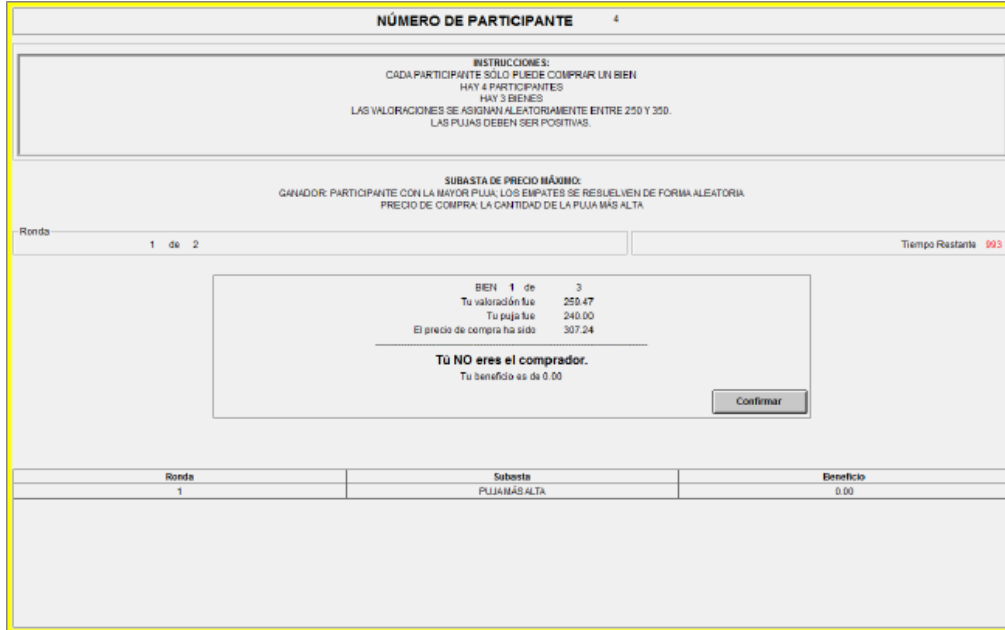


Figure S10. Non-winner of F

The third screen (Figure S11) corresponds to the winner of a Second Price auction, with its bid of 350 tokens and a Price to pay of 335 tokens, so the profit is, given the valuation of 347.59, of 12.59 tokens.

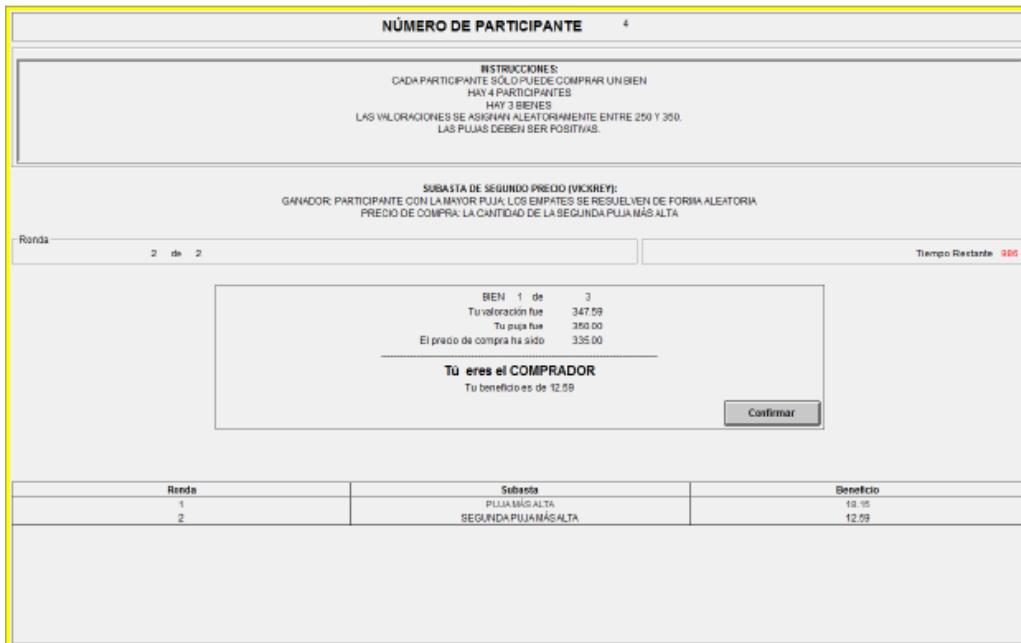


Figure S11. Winner of S

The fourth screenshot (Figure S12) corresponds to a non-winner of a Second Price auction, with a bid of 250, and a Price of 335 tokens.

NÚMERO DE PARTICIPANTE 2

INSTRUCCIONES:
CADA PARTICIPANTE SÓLO PUEDE COMPRAR UN BIEN
HAY 4 PARTICIPANTES
HAY 3 BIENES
LAS VALORACIONES SE ASIGNAN ALEATORIAMENTE ENTRE 250 Y 350.
LAS PUJAS DEBEN SER POSITIVAS.

SUBASTA DE SEGUNDO PRECIO (VICKREY):
GANADOR: PARTICIPANTE CON LA MAYOR PUJA. LOS EMPATES SE RESUELVEN DE FORMA ALEATORIA.
PRECIO DE COMPRA: LA CANTIDAD DE LA SEGUNDA PUJA MÁS ALTA.

Ronda 2 de 2 Tiempo Restante 033

BIEN 1 de 3
Tu valoración fue 270.62
Tu puja fue 250.00
El precio de compra ha sido 335.00

Tú NO eres el comprador.
Tu beneficio es de 0.00

Ronda	Subasta	Beneficio
1	PUJA MÁS ALTA	0.00
2	SEGUNDA PUJA MÁS ALTA	0.00

Figure S12. Non-winner of S

The last screen (Figure S13) shows the history of the profits for a participant that cannot bid.

NÚMERO DE PARTICIPANTE 4

INSTRUCCIONES:
CADA PARTICIPANTE SÓLO PUEDE COMPRAR UN BIEN
HAY 4 PARTICIPANTES
HAY 3 BIENES
LAS VALORACIONES SE ASIGNAN ALEATORIAMENTE ENTRE 250 Y 350.
LAS PUJAS DEBEN SER POSITIVAS.

SUBASTA DE SEGUNDO PRECIO (VICKREY):
GANADOR: PARTICIPANTE CON LA MAYOR PUJA. LOS EMPATES SE RESUELVEN DE FORMA ALEATORIA.
PRECIO DE COMPRA: LA CANTIDAD DE LA SEGUNDA PUJA MÁS ALTA.

Ronda 2 de 2 Tiempo Restante 007

BIEN 2 de 3
No puedes pujar por este bien porque ya has obtenido uno

Tu beneficio acumulado es 12.59

Ronda	Subasta	Beneficio
1	PUJA MÁS ALTA	10.15
2	SEGUNDA PUJA MÁS ALTA	12.59

Figure S13. History of profits

Obviously the numbers are factitious since they are not within the allowable range for this experiment, since the valuations will be between 30 and 90 for everyone.

As a summary, each round is composed of the following stages:

- Assignment of participants to groups

- For each group:
 - Auction for Good 1 of 3: Placement of 4 bids and assignment to the winner
 - Auction for Good 2 of 3: Placement of 3 bids and assignment to the winner
 - Auction for Good 3 of 3: Placement of 2 bids and assignment to the winner
 - Presentation of profits in tokens after each good and auction

At the end of each round, the following screen (Figure S14) will appear, asking to wait for instructions:

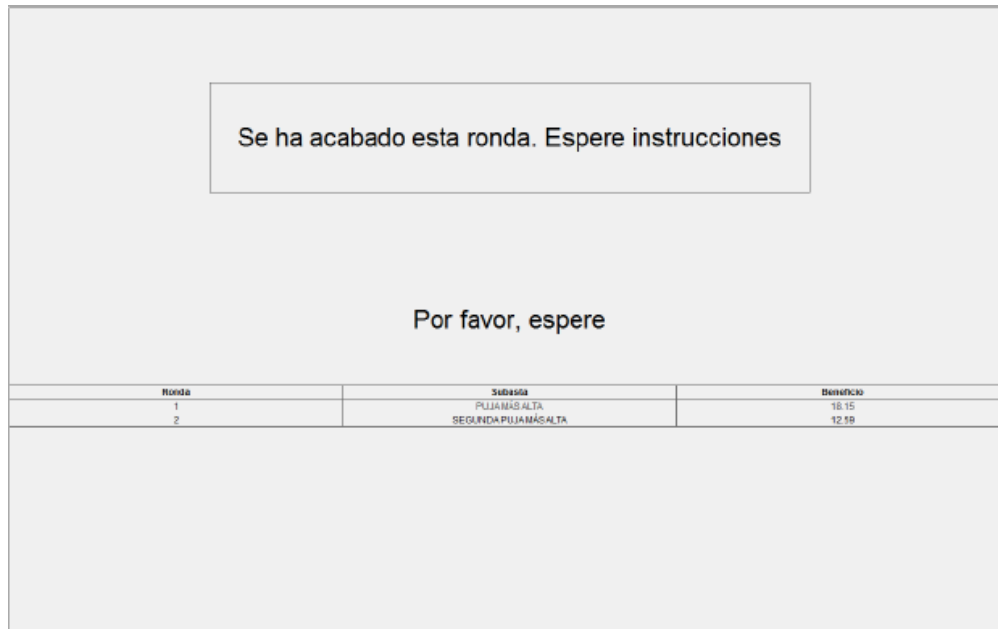


Figure S14. End of round

8 rounds will be played, the first 4 will be First Price/Second Price and the last 4 will be Second Price/First Price. A practice round will be performed first.

At the end of the experiment, after the 8 rounds, one round will be selected at random to convert the tokens obtained in that round in euros. Once the round is select, I will individually and secretely pay in euros at a conversion rate of 1 euro per 4 tokens. 2 additional euros will be payed to each individual to cover the participation.

We will now start the experiment. Please follow the instructions that I will dictate out loud.