
MULTISTAGE CONTRACTING WITH APPLICATIONS TO R&D AND INSURANCE POLICIES

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Abstract

An agent undertakes a nonobservable first-stage effort. The principal observes whether the effort results in a successful project or not. If the project succeeds, only the firm observes its interim quality, and can further improve it with a nonobservable second-stage effort. If the agent accepts penalties when the first-stage fails, moral hazard and asymmetric information do not prevent the principal from implementing her first-best outcome. However, if the agent is bounded by the maximum loss he can bear when the first-stage fails (limited liability), the principal induces the agent to exert a first-stage effort below the first-best level and a second-stage effort above the first-best level when the interim quality of his project is low. This distortion in efforts implies that the *ex post* rent left to the agent with a project of high interim quality is above the first-best level. This provides a rationale for the optimality of expanding the use of the “carrot” (second-stage rent) when the use of the “stick” (first-stage penalty) is restricted. Implications of the theory for R&D, bank, job, and insurance contracts are discussed.

1. Introduction

In the last two decades, the contract theory literature has been extensively developed around the moral hazard and asymmetric information paradigms. Investigations have first analyzed the characteristics of optimal *static* contracts

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and emphasized the trade-off efficiency versus rents.¹ However, in many economic situations, the agenda for the relationship between the principal and the agent is more complex. Most notably, the two parties may engage in a *repeated or long-term* relation. Laffont and Tirole (1988) were the first to address the issue of dynamic contracting under asymmetric information. Their paper analyzes a situation where the principal delegates the realization of a project to the agent twice, and studies the optimal incentive scheme under commitment, commitment and renegotiation, and no commitment.²

This paper studies a related multiperiod contracting problem. We consider an agent who undertakes a nonobservable first-stage effort to obtain an interim output or learn his ability to perform a task. The principal observes if this output or ability is below or above the threshold level where it is desirable to disrupt the relation. However, only the agent learns the exact output or ability. Then, if the relation is continued, the agent undertakes a nonobservable second-stage effort that affects the final output or task performance.³ This general setting captures, for example, the dynamics of job contracts within training periods. The effort exerted during the on-the-job training period allows the individual to learn his productivity within the firm but also partly informs the employer about the agent's ability. The decision whether to retain the individual will depend on the information obtained by the employer. If the agent is hired, his effort to perform efficiently in the job combined to his (privately known) productivity determines his overall performance.⁴ To sum up, our setting is characterized by multiperiod effort, and partial information

¹See, e.g., Baron and Myerson (1982) for optimal contracts under adverse selection, Picard (1987) for an analysis of incentive schemes under both moral hazard and adverse selection, and Laffont and Tirole (1993) for a broad overview of developments in contract theory applied to procurement and regulation.

²There is also a large literature on the problem of repeated moral hazard that addresses the problem of commitment (see Chiappori et al. (1994) for a survey).

³Dynamic incentive contracts in the presence of asymmetric information and moral hazard in all stages have been studied. Models show how the information revealed in an early period should be used optimally in the subsequent periods (see Laffont and Tirole 1993, Lewis and Sappington 1997). By contrast with these investigations, we assume that both agent and principal share the same information in the early stage.

⁴Lewis and Sappington (1993) analyze a job contracting model where an agent chooses his level of education in the first-stage (moral hazard) and learns his productivity on the job. This information is partially revealed to the employer who designs the second period job contract. In the second-stage, the agent decides whether to accept the short-term contract and produces in case of acceptance. The structure of information is very similar to ours. However and contrary to this analysis, we assume that the principal designs a long-term contract contingent on all the information she may possess in the future, which is accepted or not in the first period. Besides, Lewis and Sappington (1993) focus on the relationship between the optimal second-period contract and the outside option of the agent, which can be type dependent. This feature is absent in the present model.

revelation at an interim stage.⁵ One should note right away that the payoffs of the long-term contract offered by the principal will be made contingent on the observed outcome at the end of the first-stage. However, in some applications, there might be some restrictions in the maximum “punishment” that the principal can inflict to the agent when the result of the first-stage is unsatisfactory (independently of whether the agent is willing to accept such contract).⁶ Hence, limited liability at the interim stage is a crucial factor in this analysis.⁷

Even if examples such as dynamic job contracts fit our framework, for expositional convenience, we focus instead on a model of a publicly financed Research and Development activity (R&D). More concretely, a firm (he) exerts a nonobservable first-stage research effort that affects his probability of obtaining a prototype or innovation. The regulator (she) observes whether the research stage has been successful or not but, in the former case, only the firm observes the exact quality of the innovation. If the innovation is successful, the project is continued and the firm exerts a nonobservable second-stage development effort to make the invention usable. In this precise framework, we characterize the optimal contract offered by the regulator to the firm at the beginning of the game, given the double moral hazard and the anticipated future asymmetric information.

As a benchmark, we assume no limited liability in the first-stage, which means that the firm can commit to receive a negative rent if his innovation fails. In this case, the firm accepts the contract if he is rewarded with (i) nonnegative second-period rents in every state of the world, and (ii) nonnegative overall expected rents. The second period problem is the standard regulation model analyzed by Laffont and Tirole (1986): in order to induce effort, the efficient type (i.e., the firm with a high-quality innovation) must obtain some second-stage rents, which are proportional to the effort of the inefficient type (the firm with a low-quality innovation). However, we show that the regulator can obtain her first-best outcome (socially optimal efforts in the two stages and no expected rents left to the firm). The idea is simply that by “punishing” the firm sufficiently when the first (research) stage fails,

⁵Riordan and Sappington (1989) build also on a similar structure of information. The authors consider a multistage model of defense procurement with moral hazard in the first-stage and partial information revelation at the interim stage. The analysis focuses on short-term second-period contracts with either the agent who participates in the first-stage or a second source.

⁶For example, the principal may choose not to hire an agent if the training period reveals a low capacity. However, it is hard to conceive the enforceability of a contract where the employee would pay the employer whenever he is fired, even if the former is willing to subscribe it.

⁷The effects of limited liability on incentive contracts have been analyzed in the literature in various static settings. It is modeled either as an upper bound on the penalty (e.g., Baron and Besanko 1984), or a lower bound on the utility (e.g., Sappington 1983). In this paper, we choose the first technique.

the regulator can recoup the expected rent that he will be forced to leave in the second (development) stage if the innovation succeeds. The firm is willing to accept such contract because, on average, he obtains zero rents. This result is interesting but not very surprising; it replicates the findings of Harris and Raviv (1979) or Sappington (1983), who show that asymmetric information does not prevent the principal from implementing her first-best outcome as long as the parties share the same information at the contracting stage and the agent has no interim limited liability.

Given this benchmark setting, we then turn to the main contribution of the paper: the characterization of the optimal contract under limited liability, i.e., when the firm is bounded by the maximum loss he can bear at the end of the first-stage.⁸ Proposition 2 shows that, under limited liability, the regulator distorts downward (i.e., below the first-best level) the research effort of the firm and upward (i.e., above the first-best level) the development effort of the inefficient type firm. The firm gets expected positive rents. Furthermore, as the limited liability constraint becomes more stringent, the optimal second-best contract specifies a lower first-stage effort, a higher second-stage effort for the inefficient type, and more rents to the firm (Proposition 3).⁹

The intuition for this result is the following. First, the research effort is encouraged by increasing the first-stage transfers when the innovation is successful. Since the (negative) transfers when the innovation fails are bounded, this means that the positive intertemporal expected rents must be granted to the firm. In order to decrease these rents, the regulator reduces the reward for success, which induces a suboptimal first-stage effort level. This is simply the standard trade-off efficiency versus rents applied to this particular dynamic setting. Second, as usual in incentive schemes, the second-stage rents of the efficient type are increasing in the effort of the inefficient type. Therefore, by inducing a development effort for the inefficient type above the first-best level, the regulator is implicitly increasing the *ex post* second-stage rents when the innovation succeeds. As a result, the firm is more willing to incur a high first-stage effort to obtain the innovation (and enjoy these rents) even if he is not explicitly rewarded for that. In other words, the regulator has two mechanisms to foster the first period effort: transfer in case of success and second-stage rents for the efficient type. The second mechanism is relatively more interesting: it requires an above first-best development effort, which is costly but also generates some benefits to the regulator.

⁸Note that limited liability is just an extreme form of modeling risk-aversion: formally it is equivalent to risk-neutrality above a certain threshold and infinite risk-aversion below it.

⁹The fact that limited liability can induce results qualitatively different from the traditional result of second best contracting has also been observed by Lawarrée and Van Audenrode (1996). The authors consider a static adverse selection model and show that the efficient type can be induced to produce less than the inefficient type when output is imperfectly observed.

To sum up, when the regulator's use of the "stick" (first-stage negative transfers if the innovation fails) is restricted due to limited liability, she expands the use of the "carrot" (second-stage rents) in order to induce additional effort. This result departs from standard results in contract theory, where the effort of the inefficient type is always below the first-best level.¹⁰ It has also some interesting consequences. Most notably, the paper implies that, under limited liability, long-term contracts should contain the following two features. First, a training or evaluation period, after which the principal (employer, regulator, insurance company, bank, etc.) decides whether to continue or disrupt her relation with the agent (employee, firm, policy holder, individual, etc.). During this stage, the agent obtains more information than the principal about the value of the relation. Second, an incentive contract in case of continuing the relation that specifies *large inequalities* (both in terms of efforts and rewards) in the treatment of agents with different private information parameters.

Last, it is worth noting that some authors have already developed models where efforts (or quantity produced) are distorted upward in the optimal second-best contract. However, the reasons are of a very different nature. In Lewis and Sappington (1989) and Jullien (2000), e.g., the key for such a conclusion is that the agent's outside opportunity is not constant but rather a function of his private information parameter. As a result, the incentive compatibility constraint binds for the most efficient type and the rents are decreasing in the agent's efficiency level. Khalil (1997) studies the optimal contract in the Baron and Myerson (1982) framework when a principal cannot commit to an audit policy. The purpose of the contract is to provide incentives for the agent to comply as well as for the principal to audit: in equilibrium the principal threatens the agent with an audit and leaves him with no rent. In this setting, the key tradeoff is efficiency versus noncompliance instead of rent versus efficiency. By increasing the output above the first-best level, the principal convinces the agent that he has a higher stake in an audit and induces more compliance. Finally, Khalil and Lawarrée (2001) show that when the principal can monitor either the input (effort) or the output, she finds it profitable to choose the monitoring variable *ex post* in order to use the agent's fear of getting caught on the wrong foot. This might induce an agent with low type to overproduce when the asymmetry of information is sufficiently large.

The paper is organized as follows. Section 2 develops the basic model. Section 3 determines the optimal regulatory scheme in two benchmark cases: complete information (Section 3.1) and incomplete information with no limited liability (Section 3.2). Section 4 analyzes the case of incomplete information and limited liability. It states the main result of the paper (Section 4.1)

¹⁰The paper also discusses the optimal contract under no commitment, which in fact is quite trivial. It also shows that the possibility of renegotiating the long-term contract would not modify the results presented.

and performs some comparative statics (Section 4.2). Section 5 discusses some potential applications, namely R&D, bank, job, and insurance contracts. Finally, Section 6 concludes.

2. The Model

We analyze a multistage contracting problem. For expositional purposes, in the main body of the text we concentrate on the relation between a regulator and an R&D firm. However, in section 5 we argue that this formal framework is also suitable for the analysis of multistage insurance, banking, and job contracts.

We consider a risk-neutral regulated R&D firm (the agent, “he”). This firm can engage in a project that requires first a research effort (to obtain an innovation) and then a development effort (to finalize the product). We assume without loss of generality that the reservation utility of the firm is 0. The social value of the project S is fixed and sufficiently large (this ensures the concavity of the problem). The regulator (the principal, “she”) is also risk-neutral. The two-stage relationship between the regulator and the firm is governed by an extensive long-term contract with the following characteristics: (i) the regulator makes a take-it-or-leave-it offer to the firm and (ii) she proposes an optimal incentive scheme at the beginning of the first period (this means in particular that she has no legal right to dissociate herself from the firm at any moment of their relationship).

2.1. First-Stage: Research

During the first-stage, the firm chooses a *research effort* e_1 . This effort affects both the probability of obtaining an innovation and, if there is such an innovation, its “level” or “efficiency” or “quality.” More specifically, we assume that the following events may occur at the research stage: the innovation succeeds ($\gamma = \beta$) or fails ($\gamma = 0$). In case of a successful innovation, it can be a high-cost innovation ($\beta = \bar{\beta}$) or a low-cost innovation ($\beta = \underline{\beta}$), where β refers to the interim cost of the future project and $\Delta\beta \equiv \bar{\beta} - \underline{\beta} > 0$.¹¹ The regulator observes if there is an innovation or not ($\gamma = \beta$ or $\gamma = 0$). However, if the innovation succeeds, only the firm observes its level $\beta \in \{\bar{\beta}, \underline{\beta}\}$.¹²

We denote by $\pi(e_1)$ the probability of obtaining an innovation and, conditional on the innovation being successful, by $v(\underline{\beta} | e_1)$ its probability of being efficient (so $v(\bar{\beta} | e_1) = 1 - v(\underline{\beta} | e_1)$ is the conditional probability of the innovation being inefficient). Therefore, the *ex ante* probabilities of no

¹¹A “high-cost” refers to an “inefficient” innovation and a “low-cost” refers to an “efficient” one. This terminology is borrowed from Laffont and Tirole (1986).

¹²For example, a prototype has been successfully built but the regulator does not have sufficient technical information to determine its quality.

innovation, high-cost innovation, and low-cost innovation are, respectively, $1 - \pi(e_1)$, $\pi(e_1)(1 - v(\underline{\beta} | e_1))$, and $\pi(e_1)v(\underline{\beta} | e_1)$.

The effort affects positively both the probability of innovating and, in case of success, the probability of the innovation being efficient. We also assume decreasing returns in effort, and the probability of obtaining an innovation increases more rapidly than the *ex ante* probability of obtaining an innovation of high type. These assumptions are presented in Assumption 1.

ASSUMPTION 1:

- (i) $\pi(e_1)$ is increasing and concave in e_1 . Moreover, $\pi'''(e_1) \leq 0$.¹³
- (ii) $v(\underline{\beta} | e_1)$ is increasing in e_1 .
- (iii) $\pi(e_1)v(\underline{\beta} | e_1)$ is such that $\frac{d}{de_1}[\pi(e_1)v(\underline{\beta} | e_1)] < \pi'(e_1)$.

Note that given (i) and (ii) in Assumption 1, the *ex ante* probability of obtaining an innovation of high type, that is, $\pi(e_1)v(\underline{\beta} | e_1)$ is increasing in effort.¹⁴ As it is well known, moral hazard problems are easily plagued by nonconcavities. Therefore, in order to ensure the concavity of the overall maximand, we will further assume that the *ex ante* probability of obtaining an innovation is “sufficiently” concave in the firm’s research effort.

ASSUMPTION 2: $\pi(e_1)v(\underline{\beta} | e_1)$ is such that $\frac{d^2}{de_1^2}[\pi(e_1)v(\underline{\beta} | e_1)] < \pi''(e_1)$ and $\frac{d^3}{de_1^3}[\pi(e_1)v(\underline{\beta} | e_1)] < 0$.

Note that Assumptions 1 and 2 are compatible if the unconditional probability of an innovation $\pi(e_1)$ is sufficiently increasing in effort.¹⁵

2.2. Second-Stage: Development

At the end of the first-stage, the regulator observes whether an innovation has been achieved or not. If the innovation fails, then the project is stopped.¹⁶ If the innovation succeeds, the firm decides whether to develop the project in the second-stage. If he does not quit, by exerting a *development effort* e_2 , the firm reduces the interim cost of the project $\beta (\in \{\underline{\beta}, \bar{\beta}\})$ by an amount e_2 . Thus, the final cost becomes $C = \beta - e_2$.

¹³The Assumption $\pi'''(e_1) \leq 0$ is only a technical (sufficient) condition that makes the analysis more tractable. All the results hold as long as $\pi'''(e_1)$ is not “too positive.”

¹⁴As an example, $\pi(e_1) = e_1$ and $v(\underline{\beta} | e_1) = \frac{e_1}{1+e_1}$ satisfy the prescriptions in Assumption 1.

¹⁵Assumption 2 is not necessary for our main results to hold. It only guarantees that both the welfare and the expected utility of the firm are strictly concave in e_1 . In particular, the results continue to hold if $\frac{d^2}{de_1^2}[\pi(e_1)v(\underline{\beta} | e_1)] - \pi''(e_1)$ and $\frac{d^3}{de_1^3}[\pi(e_1)v(\underline{\beta} | e_1)]$ are not “too” positive.

¹⁶The results obtained in the paper hold also if we assume instead that the firm discovers a low-quality innovation (instead of failing) and the regulator decides whether to terminate the project or not. This will be discussed later on.

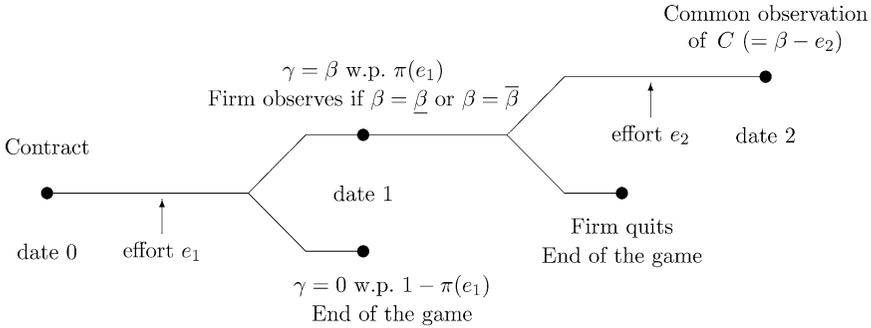


Figure 1: Timing

We assume that exerting R&D efforts (e_1 and e_2) are costly for the firm. We denote by $\psi_1(e_1)$ and $\psi_2(e_2)$ the disutilities of effort, expressed in monetary units. These functions satisfy the following assumption.

ASSUMPTION 3: $\psi'_i(e_i) > 0$, $\psi''_i(e_i) > 0$, $\psi'''_i(e_i) \geq 0 \forall i = 1, 2, \forall e_i > 0$. Besides, $\psi_i(0) = 0$, $\psi'_i(0) = 0 \forall i = 1, 2$, and $\lim_{e_2 \rightarrow \beta} \psi_2(e_2) = +\infty$.

By assuming $\psi'''_i \geq 0$, we can (without loss of generality) restrict our attention to deterministic schemes. Also, $\lim_{e_2 \rightarrow \beta} \psi_2(e_2) = +\infty$ ensures that the overall cost of the project will be positive. We assume that the regulator observes C . However, she cannot disentangle its various components (interim cost β and development effort e_2).¹⁷

The timing of the relationship between the regulator and the firm can be summarized as given in Figure 1.

2.3. Utilities and Incentive Contract

We take the accounting convention that the (observable) cost C is reimbursed to the firm. However, in addition to the reimbursement of this cost, the regulator must offer a monetary transfer t to the firm in order to induce him to accept the contract and incur the disutility of effort (the firm's reservation utility is normalized to 0). Following the standard regulation literature, we assume first that each unit of money spent by the regulator is raised through distortionary taxes and cost $(1 + \lambda)$ units to the society, and second that the regulator is "benevolent," so her objective is to maximize the total surplus of the society. We also assume that both the regulator and the firm have a discount factor equal to 1. Call U the firm's expected utility, V the consumers'

¹⁷As the reader can notice, this second-stage is identical to the standard regulation problem studied by Laffont and Tirole (1986).

net expected surplus, and $W = U + V$ the *ex ante* social welfare of an utilitarian regulator. We have:

$$U = t - \psi_1(e_1) - \pi(e_1)\psi_2(e_2) \tag{1}$$

$$V = \pi(e_1)S - (1 + \lambda)[\pi(e_1)(v(\underline{\beta} | e_1)(\underline{\beta} - e_2) + v(\bar{\beta} | e_1)(\bar{\beta} - e_2) + t)] \tag{2}$$

$$W = \pi(e_1)S - (1 + \lambda)[\psi_1(e_1) + \pi(e_1)(\psi_2(e_2) + v(\underline{\beta} | e_1)(\underline{\beta} - e_2) + v(\bar{\beta} | e_1)(\bar{\beta} - e_2))] - \lambda U. \tag{3}$$

Note that the information is symmetric when the regulator offers the contract since, at this stage, the firm does not know what will be his efficiency parameter β .¹⁸ Yet, as depicted in Figure 1, the regulator anticipates a moral hazard problem at date 0 (the firm’s research effort cannot be monitored) and a moral hazard and adverse selection problem at date 1 if the innovation succeeds (the interim cost will be privately known by the firm and his development effort cannot be monitored). She will then offer a contract to induce optimal R&D efforts (e_1 and e_2) at the expense of minimum transfers, which are socially costly given $\lambda > 0$. Section 3 characterizes the optimal contract in the benchmark cases of full information (Section 3.1) and incomplete information without limited liability (Section 3.2). The more interesting case of incomplete information and limited liability is studied in Section 4.

3. Benchmark Case: Regulation Without Limited Liability

3.1. Regulation Under Full Information

Suppose that the regulator observes the interim cost β when the innovation is successful and can monitor the research effort e_1 .¹⁹ In this full information situation, the regulator (i) imposes the effort levels e_1 and e_2 that maximize his social welfare function W and (ii) rewards the firm with a transfer t that ensures his acceptance of the contract (i.e., a contract that satisfies the firm’s participation constraint). Formally, the regulator’s maximization problem \mathcal{P}_{FI} is given by

$$\begin{aligned} \mathcal{P}_{FI} : \max_{\{e_1, e_2, t\}} W \\ \text{s.t. } U \geq 0. \end{aligned}$$

¹⁸Sappington (1982) assumes that the firm possesses better information not only on the *ex post* realization of a random state of nature but also on the *ex ante* distribution of this state of nature. The optimal policy is a menu of contracts that guarantees appropriate self-selection between contracts (i.e., induces the firm to choose the contract that corresponds to the true distribution) and appropriate self-selection within each contract (i.e., induces the firm to choose the action corresponding to the true state of nature). Following this route would not modify our results qualitatively.

¹⁹Given perfect observability of C , the regulator can deduce the development effort e_2 from the observation of the interim cost β . Therefore, e_2 does not need to be monitored.

From (3), one can immediately notice that giving rents to the firm is socially costly. The optimal solution to the problem \mathcal{P}_{FI} will then have the following characteristics: (i) the transfer t is adjusted so that the firm gets no rents ($U = 0$); (ii) if an innovation is realized, the firm’s marginal cost of second-stage (development) effort must be equal to the marginal cost-savings²⁰; (iii) the firm’s marginal cost of first-stage (research) effort must be equal to the *ex ante* marginal social benefit (both in terms of probability of innovating and probability of the innovation being efficient). These prescriptions are summarized in Lemma 1.

LEMMA 1: *Under Assumptions 1–3, the solution (e_1^*, e_2^*, t^*) to the maximization problem under full information \mathcal{P}_{FI} is such that*

- (i) $1 = \psi_2'(e_2^*);$
- (ii) $\pi'(e_1^*) [\frac{S}{1+\lambda} - v(\beta | e_1^*)\beta - v(\bar{\beta} | e_1^*)\bar{\beta} + e_2^* - \psi_2(e_2^*)] + \pi(e_1^*) \frac{\partial v(\beta | e_1)}{\partial e_1} |_{e_1^*} \Delta\beta = \psi_1'(e_1^*);$
- (iii) $t^* = \psi_1(e_1^*) + \pi(e_1^*)\psi_2(e_2^*);$

i.e., socially optimal effort levels (e_1^, e_2^*) , no expected rents ($U = 0$), and no ex post rents.*

The function W evaluated at (e_1^*, e_2^*, t^*) determines the social welfare in the first-best scenario of complete information. It thus provides a useful benchmark for comparison with the incomplete information cases under unlimited and limited liability. From part(ii) of Lemma 1, we can easily check that the optimal first-period effort is a decreasing function of the shadow cost of public funds (formally, $\partial e_1^*/\partial \lambda < 0$). It reflects the idea that as λ increases, the regulator has fewer incentives to encourage research effort because rewarding the firm for obtaining an innovation is socially more costly.

3.2. Regulation Under Incomplete Information

The analysis becomes more interesting when the regulator cannot monitor the firm’s R&D efforts and she does not observe the efficiency level of a successful innovation (formally, she can distinguish between failure ($\gamma = 0$) and success ($\gamma = \beta$) of an innovation but not between a low-cost ($\beta = \underline{\beta}$) and a high-cost ($\beta = \bar{\beta}$) successful innovation). Her optimal strategy is then to offer an incentive scheme based on the observed variables γ and C . From the revelation principle, the most general regulatory mechanism can be written as a revelation mechanism:

$$\{t_S, t_F, t(\tilde{\beta}), C(\tilde{\beta})\},$$

which specifies for each contingency of the first-stage “success” ($\gamma = \beta$) and “failure” ($\gamma = 0$):

²⁰Note that $C = \beta - e_2$, so the marginal cost-savings is equal to 1 independently of the interim cost β .

- A transfer t_S for the first-stage in case of success;
- A transfer t_F for the first-stage in case of failure; and for each announced efficiency $\tilde{\beta} (\in \{\underline{\beta}, \bar{\beta}\})$ when the innovation has succeeded:
- A net transfer $t(\tilde{\beta})$ rewarding the second-period activity;
- A cost target $C(\tilde{\beta})$.

Naturally, the firm only accepts at date 0 contracts that generate an expected intertemporal payoff greater than his reservation utility (normalized to 0). Also, after the transfers t_S or t_F have been made at date 1, the firm can quit the relationship even if his innovation has been successful, and obtain his second-stage reservation utility (also normalized to 0).²¹ This will be reflected in the participation constraints of the firm. The key issue is to determine whether the firm is restricted by the minimum payoff he can accept in some states of nature or not. In this benchmark section, we will assume that he is not.

ASSUMPTION 4a: *The firm can accept any set of transfers that satisfy his Individual Rationality constraints. In particular, there are no exogenous constraints on t_S and t_F .*

Given the previously defined incentive scheme, the firm’s intertemporal expected utility when he reports his true type in the second-stage can be rewritten as follows:²²

$$\begin{aligned}
 U_{AI}(e_1) &= \pi(e_1)t_S + (1 - \pi(e_1))t_F - \psi_1(e_1) \\
 &\quad + \pi(e_1)v(\underline{\beta} | e_1)[t(\underline{\beta}) - \psi_2(\underline{\beta} - C(\underline{\beta}))] \\
 &\quad + \pi(e_1)v(\bar{\beta} | e_1)[t(\bar{\beta}) - \psi_2(\bar{\beta} - C(\bar{\beta}))]. \tag{4}
 \end{aligned}$$

Denoted by $u_2(\beta, \tilde{\beta})$, the firm’s second-stage utility when the innovation has succeeded, he announces an efficiency parameter $\tilde{\beta}$ and his true efficiency is β . We have

$$u_2(\beta, \tilde{\beta}) = t(\tilde{\beta}) - \psi_2(e_2(\beta, \tilde{\beta})). \tag{5}$$

Using (4) and (5), we can determine the individual rationality (or participation) constraints of the firm. These are given by

$$U_{AI}(e_1) \geq 0, \tag{IR}_1$$

$$u_2(\underline{\beta}, \underline{\beta}) \geq 0, \tag{IR}_2$$

$$u_2(\bar{\beta}, \bar{\beta}) \geq 0, \tag{IR}_2$$

²¹The timing of the model implies a zero (limited) liability (in the sense of Sappington, 1983) for the second stage.

²²Note that the regulator faces a pure adverse selection problem in stage 2, since $e_2(\beta) = \beta - C(\beta)$.

where (\underline{IR}_1) states that the firm’s intertemporal utility must be nonnegative, and (\underline{IR}_2) and (\overline{IR}_2) state that the second-stage utility of a firm with a successful innovation who reveals truthfully his type must be nonnegative, independently of his efficiency level.

Naturally, the incentive contract must also induce truthful revelation of the firm’s type at stage two. These incentive compatibility constraints are given by

$$u_2(\underline{\beta}, \underline{\beta}) \geq u_2(\underline{\beta}, \bar{\beta}) \tag{IC_2}$$

$$u_2(\bar{\beta}, \bar{\beta}) \geq u_2(\bar{\beta}, \underline{\beta}). \tag{IC_2}$$

Furthermore, the firm freely chooses his first-stage effort e_1 so as to maximize his expected utility. Let us denote \tilde{e}_1 the equilibrium effort. It must satisfy the following moral hazard constraint:

$$\tilde{e}_1 = \arg \max_{e_1} U_{AI}(e_1). \tag{MH}$$

Finally, the social welfare function W_{AI} that the regulator maximizes can be rewritten as

$$\begin{aligned} W_{AI} = & \pi(e_1)S - (1 + \lambda)\pi(e_1)(v(\underline{\beta} | e_1)[\psi_2(\underline{e}_2) + \underline{\beta} - \underline{e}_2] \\ & + v(\bar{\beta} | e_1)[\psi_2(\bar{e}_2) + \bar{\beta} - \bar{e}_2]) \\ & - (1 + \lambda)\psi_1(e_1) - \lambda U_{AI}(e_1), \end{aligned} \tag{6}$$

where \underline{e}_2 and \bar{e}_2 represent the second-stage efforts of a firm with interim costs $\underline{\beta}$ and $\bar{\beta}$, respectively. At this stage, we can finally determine the regulator’s maximization problem \mathcal{P}_{AI} in the asymmetric information case without limited liability. It is given by:

$$\begin{aligned} \mathcal{P}_{AI} : & \max_{\{t_s, t_f, t(\beta), C(\beta)\}} W_{AI} \\ \text{s.t.} & \quad (\underline{IR}_1), (\underline{IR}_2), (\overline{IR}_2) \\ & \quad (\underline{IC}_2), (\overline{IC}_2), (\text{MH}) \end{aligned}$$

The main characteristics of the optimal solution to this problem are the following.

PROPOSITION 1: *Under Assumptions 1–4a, the solution to the maximization problem under incomplete information and no limited liability \mathcal{P}_{AI} is characterized by*

- (i) *The first-best effort levels ($e_1 = e_1^*$ and $\underline{e}_2 = \bar{e}_2 = e_2^*$);*
- (ii) *No expected rents ($U_{AI}(e_1^*) = 0$).*

This first-best contract is obtained with the following transfers $(t_F^*, t_S^*, t^*(\bar{\beta}), t^*(\underline{\beta}))$:

$$\begin{aligned}
 t_F^* &= \psi_1(e_1^*) - \pi(e_1^*)v(\underline{\beta} | e_1^*)\phi(e_2^*) \\
 &\quad - \frac{\pi(e_1^*)}{\pi'(e_1^*)} \left[\psi_1'(e_1^*) - \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1} \Big|_{e_1^*} \phi(e_2^*) \right] (< 0) \\
 t_S^* &= \psi_1(e_1^*) - \pi(e_1^*)v(\underline{\beta} | e_1^*)\phi(e_2^*) \\
 &\quad + \frac{1 - \pi(e_1^*)}{\pi'(e_1^*)} \left[\psi_1'(e_1^*) - \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1} \Big|_{e_1^*} \phi(e_2^*) \right] \\
 t^*(\bar{\beta}) &= \psi_2(e_2^*) \quad \text{and} \quad t^*(\underline{\beta}) = \psi_2(e_2^*) + \phi_2(e_2^*)
 \end{aligned}$$

where $\phi(e_2^*) = \psi_2(e_2^*) - \psi_2(e_2^* - \Delta\beta)$.

Proof: see Appendix A1.

Since the firm can break the relation at the end of the first stage, the second-stage is a standard adverse selection problem. As usual, the Incentive Compatibility constraint of the efficient type ($\underline{\text{IC}}_2$) and the Individual Rationality constraint of the inefficient type ($\bar{\text{IR}}_2$) are binding. Thus, the optimal contract must leave second-stage rents to the efficient type. These rents are increasing in the effort of the inefficient type (formally, $u_2(\beta, \beta) = \phi(\bar{e}_2)$). In the optimal static contract, the usual trade-off efficiency versus rents dictates that the regulator should induce the inefficient type to select a sub-optimal effort ($\bar{e}_2 < e_2^*$) in order to decrease the rents of the efficient type ($\phi(\bar{e}_2) < \phi(e_2^*)$). However, given our Multistage contract, the regulator can obtain first-best second-stage efforts for both types ($e_2 = \bar{e}_2 = e_2^*$) at no cost: it suffices to scale down first-period transfers t_F^* and t_S^* in an amount proportional to the expected second-period rents. Moreover, by penalizing the firm sufficiently (but not infinitely) in case of failure to innovate, the regulator can also induce the choice of first-best first-stage efforts ($e_1 = e_1^*$), i.e., the moral hazard constraint (MH) can also be satisfied at no social cost.²³ Note that this optimal mechanism always requires the use of negative transfers when the innovation fails ($t_F^* < 0$), which stresses the importance of Assumption 4a.

Although interesting, this result is not entirely surprising. As shown first by Harris and Raviv (1979) and further developed by Sappington (1983), if the principal and the agent share common beliefs when the contract is signed, they can write an enforceable long-term contract and if punishments are not bounded, then the principal can always obtain his first-best outcome.

Finally, if the cost of transferring public funds is sufficiently small ($\lambda < \bar{\lambda}$), the effort is socially valuable, and the regulator promotes it by offering a

²³If the firm has a discount factor $\delta < 1$, the results remain unchanged (first-best efforts and no expected rents) but expressions for the first-stage transfers are slightly modified. In particular, we have $t_S(\delta) > t_S(1)$ and $t_F(\delta) < t_F(1)$.

higher payoff in case of success than in case of failure ($t_S^* > t_F^*$). By contrast, when this cost is above a certain threshold ($\lambda > \bar{\lambda}$), encouraging effort is so costly for society that the regulator strictly prefers to discourage it ($t_S^* < t_F^*$). To our view, this second case is more a theoretical curiosity than a plausible reality. Therefore, in the remaining of the paper, we concentrate in the most natural case where it is socially optimal to encourage effort, i.e., $\lambda < \bar{\lambda}$.

Before analyzing the optimal contract with limited liability, two remarks are in order. First, the long-term contract described in Proposition 1 is renegotiation-proof. For instance, suppose that at the beginning of the second-stage and after the innovation being successful ($\gamma = \beta$), the principal decided to offer a new contract. The best she could do would be to propose the familiar static contract, characterized by the following second-stage efforts and rents: $\underline{e}_2 = e_2^*$, $\bar{e}_2 = \tilde{e}_2$ given by $\psi_2'(\tilde{e}_2) = 1 - \frac{v}{1-v} \frac{\lambda}{1+\lambda} \phi'(\tilde{e}_2)$, $u_2(\underline{\beta}, \underline{\beta}) = \phi(\tilde{e}_2)$, and $u_2(\bar{\beta}, \bar{\beta}) = 0$. An efficient firm would never accept this new contract because he would be rewarded with fewer second-stage rents ($\phi(\tilde{e}_2) < \phi(e_2^*)$). An inefficient firm would be indifferent between the two contracts (no second-stage rents in both cases) but since the effort is smaller in the new contract, the principal would be worse-off by offering it. The reason why parties do not renegotiate the contract is that the only additional information revealed to the principal at the end of the first-stage is whether the innovation succeeded or failed. Since the second-stage contract is only relevant when the innovation succeeds, there is in fact no extra useful information that the principal can use at that point to change the contract.

Second, suppose that the principal and the agent cannot sign a long-term contract. Their relationship is then run by two short-term contracts. In the second-stage, she proposes the contract described in the previous paragraph ($\bar{e}_2 = \tilde{e}_2$, $\underline{e}_2 = e_2^*$, $u_2(\underline{\beta}, \underline{\beta}) = \phi(\tilde{e}_2)$, $u_2(\bar{\beta}, \bar{\beta}) = 0$). Then, the regulator adjusts the first-stage transfers t_F and t_S so as to leave no expected rents to the firm. Interestingly, since the first-stage effort is an increasing function of the anticipated second-stage effort (see Lemma 1 part(ii)), in the first short-term contract the principal will also induce a suboptimal effort level ($e_1 < e_1^*$).²⁴ Overall, with short-term contracts, the firm's utility is unchanged (no expected rents) and the regulator is worse-off (suboptimal effort levels).

4. Optimal Regulation Under Incomplete Information and Limited Liability

We have shown that incomplete information does not reduce social welfare as long as the regulator can distort sufficiently the transfers in the different

²⁴The long-term interaction between the agent and the regulator is a false repeated interaction, since there is no information acquisition in the first-period. Hence ratcheting is not a concern for our analysis.

states of nature. We now study a more realistic situation in which the firm faces a limited liability constraint.

4.1. The Optimal Contract

Suppose that the firm has a common-knowledge initial wealth $k (\geq 0)$ and no possibility to borrow. This implies that the first-stage transfer cannot be below $-k$.²⁵ If $k > |t_F^*|$, this constraint is not binding, and therefore the first-best solution can be attained with the same contract as in Proposition 1.²⁶ The analysis changes when we assume that the firm's initial wealth effectively constrains the first-stage transfers between the regulator and the firm. This occurs under the following assumption.

ASSUMPTION 4b: $k < |t_F^*|$.

The regulator's maximization problem \mathcal{P}_{LL} in the asymmetric information case with limited liability then becomes

$$\begin{aligned} \mathcal{P}_{LL} : \quad & \max_{\{t_S, t_F, t(\beta), C(\beta)\}} W_{AI} \\ \text{s.t.} \quad & (\text{IR}_1), (\underline{\text{IR}}_2), (\overline{\text{IR}}_2) \\ & (\underline{\text{IC}}_2), (\overline{\text{IC}}_2), (\text{MH}) \\ & t_S \geq -k, \quad t_F \geq -k. \end{aligned}$$

In this case, and despite the possibility of transferring money from the second-stage to the first-stage, there is a trade-off between rent extraction and efficiency. The main contribution of this paper is a characterization of the optimal solution to this problem. The result is the following.

PROPOSITION 2: *Under Assumptions 1–4b, the solution to the maximization problem under incomplete information and limited liability \mathcal{P}_{LL} is characterized by*

- (i) *A first-best second-stage effort for the efficient type: $e_2 = e_2^*$; A second-stage effort above the first-best level for the inefficient type: $\bar{e}_2 = \hat{e}_2 > e_2^*$; A first-stage effort below the first-best level: $e_1 = \hat{e}_1 < e_1^*$,*
- (ii) *Positive expected rents.*

This contract is obtained with the following transfers $(\hat{t}_F, \hat{t}_S, \hat{t}(\bar{\beta}), \hat{t}(\underline{\beta}))$:

$$\begin{aligned} \hat{t}_F &= -k \quad \text{and} \quad \hat{t}_S = \frac{1}{\pi'(\hat{e}_1)} \left[\psi_1'(\hat{e}_1) - \frac{d[\pi(e_1)v(\beta | e_1)]}{de_1} \Big|_{\hat{e}_1} \phi(\hat{e}_2) \right] - k \\ \hat{t}(\bar{\beta}) &= \psi_2(\hat{e}_2) \quad \text{and} \quad \hat{t}(\underline{\beta}) = \psi_2(e_2^*) + \phi_2(\hat{e}_2). \end{aligned}$$

²⁵Note that a transfer of $-k$ means that the firm is paying k to the regulator in the first-stage.

²⁶Remember that we have assumed $\lambda < \bar{\lambda}$ so that $t_S^* > t_F^*$.

Proof: see the Appendix A2.

It is both surprising and interesting that in the optimal second-best contract the inefficient type exerts a second-stage effort *above* the first-best level. Let us explain step by step the reason for such distortion. Given limited liability, the regulator will set a transfer in case of failure as low as possible, i.e., $\hat{t}_F = -k$. By keeping the same difference in transfers between success and failure as in Proposition 1 (i.e., $\hat{t}_S - \hat{t}_F = t_S^* - t_F^*$), the regulator could still achieve the first-best efforts. However, due to the lower bound imposed in first-stage transfers, this would imply too much rent left to the firm. Therefore, in order to decrease these rents, efforts have to be distorted. The key issue is that the intertemporal expected rents of the firm are increasing in e_1 and decreasing in \bar{e}_2 . In other words, in order to decrease the transfer in case of success \hat{t}_S (and therefore reduce the expected rents), the regulator has to induce a lower first-stage effort and/or a higher second-stage effort for the inefficient type.²⁷ The first effect is the standard trade-off efficiency versus rents. The firm's response to a lower transfer in case of success is to reduce the effort e_1 in order to obtain such reward. The second effect is more subtle. By increasing the development effort of the inefficient type (\bar{e}_2), the regulator is increasing the second-stage rents of the efficient type ($\phi(\bar{e}_2)$). This encourages the firm to incur the disutility of effort in the first-stage in order to increase his chances of obtaining an innovation, and therefore reduces the need of the first-stage transfers for success.

Another way to understand the result is the following. Suppose that the firm's effort were $e_1 = \hat{e}_1$ and $\bar{e}_2 = \underline{e}_2 = e_2^*$. If the regulator induces the inefficient type to incur a second-stage effort above the first-best level ($\bar{e}_2 > e_2^*$), she increases the second-stage rents of the efficient type. As a result, the firm is willing to exert the same first-stage effort \hat{e}_1 in exchange of a smaller transfer \hat{t}_S in case of success. Overall, the regulator pays the same rents and achieves higher effort levels, which implies a higher total welfare.

Overall, in the second-stage of the game, the regulator is forced to use the "carrot" as an incentive scheme due to asymmetric information and the firm's capacity to terminate their relationship. We showed in Proposition 1 that this will not imply any welfare loss as long as the regulator is not bounded in her use of the "stick" in the first-stage. However, when the use of the "stick" is restricted (i.e., when limited liability constrains the size of the penalties that can be imposed), Proposition 2 states that the regulator's optimal reaction is to expand her use of the "carrot" to induce additional effort.

Our analysis provides normative prescriptions about the optimal shape of the contract that a benevolent regulator should offer to a firm in order to finance his R&D project with public funds. A major difficulty for the design

²⁷Technically, for the (MH) constraint to hold, the difference in transfers between success and failure must satisfy $t_S - t_F = \frac{1}{\pi'(e_1)} [\psi_1'(e_1) - \frac{d[\pi(e_1)^v(\beta_1 e_1)]}{de_1} \phi(\bar{e}_2)]$. Given the exogenous bound on t_F and Assumptions 1–3, it implies that t_S is increasing in e_1 and decreasing in \bar{e}_2 .

of efficient policies to encourage R&D is the ubiquitous presence of MH and asymmetric information.²⁸ According to our model, the contracts that penalize firms when their research activity fails are efficient (Proposition 1). However, when the firm's limited liability constrains the use of this punishment, the regulator should offer rewards above the first-best level for the development activity in order to encourage high research efforts (Proposition 2).

4.2. Comparative Statics on the Limited Liability Level

We have shown that the characteristics of the optimal contract depend crucially on the firm's level of limited liability. In order to determine the generality of our previous result, we now perform some comparative statics.

PROPOSITION 3: *Under Assumptions 1–4b, we have*

- (i) \hat{e}_1 is increasing in k and $\lim_{k \rightarrow |t_f^*|} \hat{e}_1 = e_1^*$;
 \hat{e}_2 is decreasing in k and $\lim_{k \rightarrow |t_f^*|} \hat{e}_2 = e_2^*$;
 $\underline{e}_2 = e_2^*$ for all k .
- (ii) Rents are decreasing in k and tend to 0 as k tends to $|t_f^*|$.

Proof: see Appendix A3.

The optimal mechanism entails a compensation of the distortions between the two stages. The more \hat{e}_1 is (downward) distorted from e_1^* , the more \hat{e}_2 is (upward) distorted from e_2^* . These distortions are inversely proportional to the firm's level of initial wealth k . However, one should notice that the degree of the first-stage limited liability has a *direct* effect on \hat{e}_1 (first-stage effort is decreased when the firm cannot be punished sufficiently) and an *indirect* effect on \hat{e}_2 (second stage effort of the inefficient type is increased to limit the decrease in \hat{e}_1). In particular, if withholding the transfer is the maximum penalty that can be imposed (i.e., when $k = 0$), the research effort is at the smallest possible level, which is compensated by the highest possible upward distortion in the development stage. Finally, given that limited liability constrains the punishment that the regulator can inflict, the rents enjoyed by the firm will be inversely proportional to his capacity to bear losses.

We would like to conclude the analysis of the limited liability case with some remarks. First, in Section 4.1, we have not imposed a lower bound in the *ex post* wealth of the firm at the end of the first-stage. This implies in particular that the firm's *ex post* utility when the project fails can be negative (more precisely, it is equal to $t^F - \psi(\hat{e}_1) \geq 0$). Naturally, our analysis would be identical if we rather imposed a lower bound on the firm's utility. Technically,

²⁸Sappington (1982) and Wright (1983), for example, explicitly consider the implications of asymmetric information on R&D activities.

if $k < |t_F^*| + \psi_1(e_1^*)$, then the limited liability constraint would bind and the results of Proposition 2 would apply, and if $k > |t_F^*| + \psi_1(e_1^*)$, then the constraint would be slack and the results of Proposition 1 would hold.

Second, the reader might wonder whether the effects highlighted in Propositions 2 and 3 would still be present when projects do not either succeed or fail. Consider a variant of this model where the quality of the interim innovation is either $\bar{\beta}$ or $\underline{\beta}$ (high quality) with probability $\pi(e_1)$, and it is either $\bar{\theta}$ or $\underline{\theta}$ (low quality) with probability $1 - \pi(e_1)$. To simplify, suppose also for simplicity that $\underline{\theta} > \bar{\beta}$. As long as the principal can disentangle between high and low quality, the optimal contract in the absence of limited liability is such that the first-best is achieved and the firm is penalized when the interim innovation is of low quality. If we impose a lower bound in first-stage transfers, the regulator achieves the first-best only by granting a large transfer when the quality of the interim innovation is high. With the same logic as in Section 4.1, the regulator decreases this transfer and promotes high quality final innovations by granting large rents to such innovations. Overall, similar results can be obtained when projects do not fail and are not terminated.

Third, renegotiation of the long-term contract is not an issue for the same reasons as in Section 3. Indeed, the renegotiated contract is again the standard scheme. An inefficient agent has more incentives than in Section 3 to renegotiate since the long-term contract leads him to implement a higher effort than the first-best effort and the renegotiated contract makes him implement a smaller effort compared to the first-best effort. But again, the regulator prefers that he implements the long-term effort. By contrast, the incentives to decrease the rents of the efficient type are higher in that case, but the agent would not accept to renegotiate a smaller level of rents if he continues to implement the same effort. Then, the long-term contract with limited liability is robust to renegotiation.

Fourth, suppose that the two actors cannot sign a long-term contract and instead have to rely on two short-term ones. In the second-stage, the principal offers the optimal static contract (with $\bar{e}_2 = \tilde{e}_2$ and $\underline{e}_2 = e_2^*$, as discussed at the end of Section 3). Naturally, given limited liability, she cannot recoup all the rents with the first-stage transfers. She will then distort downward the first-stage effort to solve the traditional trade-off efficiency versus rents. Since $\tilde{e}_2 < e_2^*$, the transfer in case of success needs to be sufficiently large to induce the firm to incur the disutility of effort in the first-stage and it is optimal to set $e_1 < e_1^*$. However, when k is small, the regulator cannot compensate a decrease in the first-stage effort by an increase in the second-stage effort, and e_1 is not necessarily smaller than \hat{e}_1 .²⁹

²⁹Formally, when $k \rightarrow |t_F^*|$, in which case $\hat{e}_1 \rightarrow e_1^*$, the first-stage short-term contract induces the firm to choose $e_1 < \hat{e}_1$. When $k < |t_F^*|$, we can have $e_1 \gtrless \hat{e}_1$ depending on the value of k .

5. Applications

We have shown that the dynamic nature of our problem where rents left in the second-stage can be (partially or totally) recouped in the first one has interesting implications for the optimal structure of contracts. In particular, the principal might encourage efforts above the first-best level for reasons that, to our knowledge, are novel in the contracting literature. For expositional convenience, we have focused on the case of a regulator and an R&D firm. However, we feel that our analysis can be relevant in a wide range of dynamic principal–agent relations where the first-period is subject to MH and the second to adverse selection and MH. In the next paragraphs, we informally discuss some possible applications.

5.1. Training-Before-Hiring

Managers frequently offer to their potential employees implicit long-term contracts that specify a reward scheme during the training stage, a rule according to which the employee will be retained after that period, and a salary if the individual is finally hired. Proposition 1 provides the main characteristics of the optimal contract that the manager should offer to the worker if the latter can be punished for not acquiring skills in the training period. Naturally, a more realistic approach suggests that employees are protected by laws, which substantially limits the losses they can suffer. Under such circumstances, Proposition 2 argues that managers should optimally commit to long-term contracts that specify large inequalities (both in terms of effort and rewards) between workers with high and average skills.³⁰

5.2. Insurance

As extensively documented in the economics of information literature,³¹ MH and adverse selection are the two major concerns in the design of insurance contracts. The interaction between an insurance agency and a policyholder is often governed by a dynamic relationship. For instance, individuals who subscribe an insurance policy for the first time are often forced to go first through a “test” period. During that time, they pay the insurance premium but do not enjoy the full benefits of the policy.³² Then, the company decides whether to

³⁰The reader might (correctly) argue that explicit commitments are infrequent in labor relations. In our view, commitment is informal and endogenous. For a different example of optimal incentive contracts between employer and employee, see Lewis and Sappington (1993).

³¹See Laffont (1989).

³²The Belgian Social Security provides the most extreme example we know of this type of contracts. The law requires health coverage to every individual. However, it also imposes to individuals who are being covered for the first time a 9-month period (called “stage”) in which they contribute to social security but are not entitled to any benefit.

accept the application (in which case the individual chooses among different incentive contracts) or to refuse the application. During the test period, the policyholder exerts effort either to increase his ability (e.g., his driving skills in the case of auto insurance) or to learn his type (e.g., his health status and risk in the case of medical insurance). The insurer also obtains some information and, on its basis, decides whether to proceed with the application. As shown in Propositions 1 and 2, it is optimal for the insurer to commit to long-term contracts, with a clause for penalty (e.g., by inflicting a monetary penalty or by refusing to cover losses incurred during the first period) at the end of the test period when certain conditions are met (e.g., an accident, the discovery that the individual suffers a rare disease). Finally, in the presence of limited liability issues, it is optimal for the insurer to promise high benefits after the test period for the “safest” clients (rents above the static second-best optimal contract) at the expense of the “risky” ones, who will need to exert above first-best efforts in order to be covered.

5.3. Other Applications: Credit Cards and Bank Accounts

Credit companies do not usually grant credit cards to individuals without credit history. Since the only way to build a history is by making an appropriate use of the credit card, some individuals may have a hard time entering into the system. To solve this problem, credit institutions offer long-term implicit contracts with the following characteristics. The individual is first granted a debit card. If there is no record of misuse after a trial period, the credit institution upgrades the card to include all the services and credit benefits. Banks follow a similar system with clients who open an account for the first time.

6. Concluding Remarks

This paper has characterized the optimal contract when a principal delegates to an agent the realization of a project that requires two steps to be achieved. Our main focus has been to investigate the role of limited liability on the optimal incentive scheme. The main thrust of our analysis is that, under limited liability, the optimal mechanism prescribes efforts below the first-best level in the first-stage and, as a compensation, efforts and rents above the first-best level in the second-stage. The reason for this result is simple. When the agent has no limited liability, the principal achieves the first-best contract with an optimal balance between penalties (first-stage negative transfers) and rewards (second-stage rents). The presence of limited liability constraints the use of the first tool, which encourages the use of the second one as a compensation scheme.

Although publicly financed R&D is the most natural application of our theory, we have argued that the model applies to other dynamic contracting

problems where the agent is not informed about some of his own characteristics at the beginning of the game and learns them over time. Situations of this kind occur frequently when the individual signs a contract for the very first time: he gets his first credit card, a new type of job, his driving license, and applies for insurance, etc. In these situations, the contract often includes a “training” period during which the individual is not treated the same way as other (more senior) contractors. Besides, principals promise a future substantially unequal treatment depending on the results of the training stage. These features correspond precisely to the prescriptions of our model.

Appendix

Proof of Proposition 1: Under incomplete information, the regulator’s maximization \mathcal{P}_{AI} problem is

$$\begin{aligned} \mathcal{P}_{AI} : \quad & \max_{\{t_S, t_F, t(\underline{\beta}), C(\underline{\beta})\}} W_{AI} \\ \text{s.t.} \quad & u_2(\underline{\beta}, \underline{\beta}) \geq u_2(\bar{\beta}, \bar{\beta}) + \psi_2(\bar{e}_2) - \psi_2(\bar{e}_2 - \Delta\beta) & (\underline{\text{IC}}_2) \\ & u_2(\bar{\beta}, \bar{\beta}) \geq u_2(\underline{\beta}, \underline{\beta}) + \psi_2(\underline{e}_2) - \psi_2(\underline{e}_2 + \Delta\beta) & (\bar{\text{IC}}_2) \\ & U_{AI}(e_1) \geq 0 & (\text{IR}_1) \\ & u_2(\underline{\beta}, \underline{\beta}) = t(\underline{\beta}) - \psi_2(\underline{e}_2) \geq 0 & (\underline{\text{IR}}_2) \\ & u_2(\bar{\beta}, \bar{\beta}) = t(\bar{\beta}) - \psi_2(\bar{e}_2) \geq 0 & (\bar{\text{IR}}_2) \\ & \tilde{e}_1 = \arg \max_{e_1} U_{AI}(e_1). & (\text{MH}) \end{aligned}$$

The second-stage problem can be treated separately. As usual, it can be easily checked that $(\bar{\text{IR}}_2)$ and $(\underline{\text{IC}}_2)$ bind whereas $(\underline{\text{IR}}_2)$ and $(\bar{\text{IC}}_2)$ do not bind. Therefore, $u_2(\bar{\beta}, \bar{\beta}) = 0$ and $u_2(\underline{\beta}, \underline{\beta}) = \phi(\bar{e}_2)$, with $\phi(\bar{e}_2) = \psi_2(\bar{e}_2) - \psi_2(\bar{e}_2 - \Delta\beta)$. Using (4), the expected utility of the firm can be rewritten as

$$\begin{aligned} U_{AI}(e_1) &= \pi(e_1)t_S + (1 - \pi(e_1))t_F - \psi_1(e_1) \\ &\quad + \pi(e_1)v(\underline{\beta} | e_1)\phi(\bar{e}_2) \geq 0. \end{aligned} \tag{A1}$$

Note that Assumption 2 ensures that the expected utility is concave in e_1 . From (6), it is immediate to check that $U_{AI}(e_1)$ enters negatively in the objective function. Therefore, in the absence of a limited liability constraint in the first-period, the regulator leaves no expected rent, i.e.,

$U_{AI}(e_1) = 0$. We neglect momentarily the MH constraint. Optimizing we obtain the three further optimality conditions

$$\psi'_2(\underline{e}_2) = 1 \quad \text{and} \quad \psi'_2(\bar{e}_2) = 1$$

$$\begin{aligned} \pi'(e_1) \left[\frac{S}{1 + \lambda} - v(\underline{\beta} | e_1)\underline{\beta} - v(\bar{\beta} | e_1)\bar{\beta} + e_2 - \psi_2(e_2) \right] \\ + \pi(e_1) \frac{\partial v(\underline{\beta} | e_1)}{\partial e_1} \Delta\beta = \psi'_1(e_1). \end{aligned} \tag{A2}$$

Thus, $\bar{e}_2 = \underline{e}_2 = e_2^*$ and $e_1 = e_1^*$. It remains to show that the MH constraint can be satisfied at no cost. MH can be rewritten as

$$t_S - t_F = \frac{1}{\pi'(e_1^*)} \left[\psi'_1(e_1^*) - \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1} \Big|_{e_1^*} \phi(e_2^*) \right]. \tag{A3}$$

Combining $U_{AI}(e_1^*) = 0$ and (9), we have

$$\begin{aligned} t_F^* = \psi_1(e_1^*) - \pi(e_1^*)v(\underline{\beta} | e_1^*)\phi(e_2^*) \\ - \frac{\pi(e_1^*)}{\pi'(e_1^*)} \left[\psi'_1(e_1^*) - \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1} \Big|_{e_1^*} \phi(e_2^*) \right] \end{aligned} \tag{A4}$$

$$\begin{aligned} t_S^* = \psi_1(e_1^*) - \pi(e_1^*)v(\underline{\beta} | e_1^*)\phi(e_2^*) \\ + \frac{1 - \pi(e_1^*)}{\pi'(e_1^*)} \left[\psi'_1(e_1^*) - \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1} \Big|_{e_1^*} \phi(e_2^*) \right]. \end{aligned} \tag{A5}$$

Once we have found the optimal mechanism $\{e_1^*, e_2^*, t_F^*, t_S^*, t^*(\underline{\beta}), t^*(\bar{\beta})\}$, we now prove some properties of the first-stage transfers, namely: (i) $\exists \bar{\lambda}$ s.t. $t_S^* > t_F^* \forall \lambda < \bar{\lambda}$ and $t_S^* < t_F^* \forall \lambda > \bar{\lambda}$ and (ii) $t_F^* < 0 \forall \lambda$.

- (i) Denote $G(e_1) = \psi'_1(e_1) - \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1} \phi(e_2^*)$, so (9) can be written $t_S - t_F = \frac{G(e_1^*)}{\pi'(e_1^*)}$. From (A2), it is immediate that $\partial e_1^* / \partial \lambda < 0$, which implies that $dG/d\lambda < 0$. Also from (A2), we have

$$\begin{aligned} \lim_{\lambda \rightarrow 0} G(e_1^*(\lambda)) = \pi'(e_1^*(0)) [S + e_2^* - \psi_2(e_2^*) - \bar{\beta}] \\ + [\Delta\beta - \phi(e_2^*)] \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1} \Big|_{e_1^*(0)}. \end{aligned}$$

By construction $\phi(e_2^*) = \psi_2(e_2^*) - \psi_2(e_2^* - \Delta\beta) < \Delta\beta$. In addition if S is large enough, then $S + e_2^* - \psi_2(e_2^*) - \bar{\beta} > 0$ and therefore

$\lim_{\lambda \rightarrow 0} G(e_1^*(\lambda)) > 0$. When $\lambda \rightarrow +\infty$, then $e_1 \rightarrow \underline{e}_1$ where \underline{e}_1 is given by

$$\begin{aligned} & \pi'(\underline{e}_1) \left[-v(\underline{\beta} | \underline{e}_1) \underline{\beta} - v(\bar{\beta} | \underline{e}_1) \bar{\beta} + e_2^* - \psi_2(e_2^*) \right] \\ & + \pi(\underline{e}_1) \frac{\partial v(\underline{\beta} | e_1)}{\partial e_1} \Big|_{\underline{e}_1} \Delta\beta = \psi_1'(\underline{e}_1). \end{aligned}$$

Then

$$\begin{aligned} \lim_{\lambda \rightarrow +\infty} G(e_1^*(\lambda)) &= \pi'(\underline{e}_1) \left[e_2^* - \psi_2(e_2^*) - \bar{\beta} \right] \\ &+ \left[\Delta\beta - \phi(e_2^*) \right] \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1} \Big|_{\underline{e}_1}. \end{aligned}$$

Note that $e_2^* - \psi_2(e_2^*) - \bar{\beta} < 0$ and $\Delta\beta - \phi(e_2^*) < -e_2^* + \psi_2(e_2^*) + \bar{\beta}$, which implies that $\lim_{\lambda \rightarrow +\infty} G(e_1^*(\lambda)) < 0$. As a consequence, there exists $\bar{\lambda}$ such that $t_S^* > t_F^*$ for all $\lambda < \bar{\lambda}$, and $t_S^* < t_F^*$ for all $\lambda > \bar{\lambda}$.

(ii) Rewriting (8), we have

$$\begin{aligned} & \pi'(e_1^*) \left[\frac{S}{1+\lambda} - \psi_2(e_2^*) + e_2^* - \bar{\beta} \right] \\ & - \left[\psi_1'(e_1^*) - \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1} \Big|_{e_1^*} \Delta\beta \right] = 0. \end{aligned}$$

Multiplying this expression by $\pi(e_1^*)/\pi'(e_1^*)$, we obtain

$$\begin{aligned} & \pi(e_1^*) \left[\frac{S}{1+\lambda} - \psi_2(e_2^*) + e_2^* - \bar{\beta} \right] \\ & = \frac{\pi(e_1^*)}{\pi'(e_1^*)} \left[\psi_1'(e_1^*) - \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1} \Big|_{e_1^*} \Delta\beta \right]. \end{aligned} \tag{A6}$$

Given (6), $\underline{e}_2 = \bar{e}_2 = e_2^*$, and $U_{AI}(e_1^*) = 0$, we have that at the optimum

$$\begin{aligned} \frac{W_{AI}}{1+\lambda} &= \pi(e_1^*) \frac{S}{1+\lambda} - \psi_1(e_1^*) \\ & - \pi(e_1^*) \left[\psi_2(e_2^*) - e_2^* + v(\underline{\beta} | e_1^*) \underline{\beta} + v(\bar{\beta} | e_1^*) \bar{\beta} \right]. \end{aligned}$$

Since the regulator can always shut down the project, we necessarily have $W_{AI} \geq 0$. Rearranging terms

$$\begin{aligned} W_{AI} \geq 0 &\Leftrightarrow \pi(e_1^*) \left[\frac{S}{1+\lambda} - \psi_2(e_2^*) + e_2^* - \bar{\beta} \right] \\ &\geq \psi_1(e_1^*) - \pi(e_1^*) v(\underline{\beta} | e_1^*) \Delta\beta. \end{aligned}$$

Combining (12) with the previous inequality, we get

$$\begin{aligned} & \psi_1(e_1^*) - \pi(e_1^*)v(\underline{\beta} | e_1^*)\Delta\beta \\ & - \frac{\pi(e_1^*)}{\pi'(e_1^*)} \left[\psi_1'(e_1^*) - \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1} \Big|_{e_1^*} \Delta\beta \right] \leq 0. \end{aligned}$$

From (10) and given that $\phi(e_2^*) < \Delta\beta$, we finally obtain that $t_F^* < 0$. ■

Proof of Proposition 2: Given Proposition 1, we know that in the absence of limited liability $t_F < 0$ in order to limit the agent's rents (which enter negatively in the objective function). Under Assumption 4b, there is a lower bound on transfers. The inequality $t_F \geq -k$ will always bind, since the regulator will try to minimize the rents, thus the transfers he gives to the agent. Hence, if $\lambda < \bar{\lambda}$, we have $\hat{t}_F = -k \leq \hat{t}_S$. As before, $(\underline{\text{IR}}_2)$ and $(\underline{\text{IC}}_2)$ bind whereas $(\underline{\text{IR}}_2)$ and $(\underline{\text{IC}}_2)$ do not bind. Hence, with the help of (A1), we can derive the expected utility of the firm in the limited liability case. It is given by

$$\begin{aligned} U_{LL}(e_1, \bar{e}_2) &= \pi(e_1)t_S - (1 - \pi(e_1))k \\ &\quad - \psi_1(e_1) + \pi(e_1)v(\underline{\beta} | e_1)\phi(\bar{e}_2) \geq 0. \end{aligned} \tag{A7}$$

As a result, the regulator's maximization problem \mathcal{P}_{LL} can be rewritten as

$$\begin{aligned} \mathcal{P}_{LL} : \quad & \max_{\{e_1, \bar{e}_2, e_2\}} W_{LL} \\ \text{s.t.} \quad & U_{LL}(e_1, \bar{e}_2) \geq 0 \\ & \pi'(e_1)(t_S + k) = \psi_1'(e_1) - \frac{\partial[\pi(e_1)v(\underline{\beta} | e_1)]}{\partial e_1} \phi(\bar{e}_2) \\ & t_S \geq -k. \end{aligned}$$

The objective function is simply

$$\begin{aligned} W_{LL} &= \pi(e_1)[S - (1 + \lambda)(v(\underline{\beta} | e_1)[\psi_2(e_2) + \underline{\beta} - \underline{e}_2] \\ &\quad + v(\bar{\beta} | e_1)[\psi_2(\bar{e}_2) + \bar{\beta} - \bar{e}_2])] \\ &\quad - (1 + \lambda)\psi_1(e_1) - \lambda U_{LL}(e_1, \bar{e}_2). \end{aligned}$$

The first constraint ensures that the expected utility is positive ($U_{LL}(e_1, \bar{e}_2) \geq 0$) so it is the analogue of $(\underline{\text{IR}}_1)$. The second constraint is simply (MH) when $t_F = -k$ (see (9)). The third constraint guarantees that t_S satisfies Assumption 4b (i.e., the limited liability constraint). We will call it (LL) in the rest of the proof.

CLAIM 1: *The solution to \mathcal{P}_{LL} entails $\underline{e}_2 = e_2^*$. To satisfy (MH) and (LL), the expected utility $U_{LL}(e_1, \bar{e}_2)$ is increasing in e_1 and decreasing in \bar{e}_2 .*

Proof: Effort $\underline{e}_2 = e_2^*$ follows directly from the FOC in \mathcal{P}_{LL} with respect to \underline{e}_2 ($\psi_2'(\underline{e}_2) = 1$). In order to satisfy (MH), t_S must be such that:

$$t_S + k = \frac{1}{\pi'(e_1)} \left[\psi_1'(e_1) - \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1} \phi(\bar{e}_2) \right] = h(e_1, \bar{e}_2). \quad (\text{A8})$$

Hence, using (A7), for any couple (e_1, \bar{e}_2) that satisfy (MH) the firm's rent can be written as

$$U_{LL}(e_1, \bar{e}_2) = \pi(e_1)h(e_1, \bar{e}_2) - k - \psi_1(e_1) + \pi(e_1)v(\underline{\beta} | e_1)\phi(\bar{e}_2).$$

Note that $\frac{\partial U_{LL}(e_1, \bar{e}_2)}{\partial e_1} = \pi'(e_1)h(e_1, \bar{e}_2) + \pi(e_1)\frac{\partial h(e_1, \bar{e}_2)}{\partial e_1} - \psi_1'(e_1) + \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1}\phi(\bar{e}_2)$. However, $\pi'(e_1)h(e_1, \bar{e}_2) = \psi_1'(e_1) - \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1} \times \phi(\bar{e}_2)$ and $h_1(e_1, \bar{e}_2) = \frac{1}{\pi'(e_1)} [\psi_1''(e_1) - \frac{d^2[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1^2} \phi(\bar{e}_2)] - \frac{\pi''(e_1)}{\pi'(e_1)} \times h(e_1, \bar{e}_2)$ is positive when $h(e_1, \bar{e}_2) > 0$. Then by (A8), we get

$$\frac{\partial U_{LL}(e_1, \bar{e}_2)}{\partial e_1} = \pi(e_1) \frac{\partial h(e_1, \bar{e}_2)}{\partial e_1} > 0.$$

Similarly, $\frac{\partial U_{LL}(e_1, \bar{e}_2)}{\partial \bar{e}_2} = \pi(e_1) \frac{\partial h(e_1, \bar{e}_2)}{\partial \bar{e}_2} + \pi(e_1)v(\underline{\beta} | e_1)\phi'(\bar{e}_2)$. From (A8), we get that $\frac{\partial h(e_1, \bar{e}_2)}{\partial \bar{e}_2} = -\frac{1}{\pi'(e_1)} \frac{d[\pi(e_1)v(\underline{\beta} | e_1)]}{de_1} \phi'(\bar{e}_2)$, and therefore

$$\frac{\partial U_{LL}(e_1, \bar{e}_2)}{\partial \bar{e}_2} = -\frac{[\pi(e_1)]^2}{\pi'(e_1)} \frac{\partial v(\underline{\beta} | e_1)}{\partial e_1} \phi'(\bar{e}_2) < 0.$$

Consequently, the rent is increasing in e_1 and decreasing in \bar{e}_2 . Besides, for a given indifference curve of level \bar{U}_{LL} :

$$\frac{d\bar{e}_2}{de_1} = -\frac{\partial U_{LL}(e_1, \bar{e}_2)/\partial e_1}{\partial U_{LL}(e_1, \bar{e}_2)/\partial \bar{e}_2} > 0.$$

Moreover, it is easy to show that if $\pi'''(\cdot) < 0$ and $\frac{\partial^3[\pi(e_1)v(\underline{\beta} | e_1)]}{\partial e_1^3} < 0$, then $h(e_1, \bar{e}_2)$ is convex with respect to e_1 , concave with respect to \bar{e}_2 , and with a positive cross derivative. These are sufficient conditions that ensure the convexity of the indifference curves.

CLAIM 2: *The solution to \mathcal{P}_{LL} entails $e_1 < e_1^*$ and $\bar{e}_2 > e_2^*$.*

The welfare can be expressed as follows

$$W_{LL} = g(e_1, \bar{e}_2) - \lambda U_{LL}(e_1, \bar{e}_2),$$

where, given that $\underline{e}_2 = e_2^*$, the function $g(e_1, \bar{e}_2)$ is

$$g(e_1, \bar{e}_2) = \pi(e_1) [S - (1 + \lambda)(v(\beta | e_1) [\psi_2(e_2^*) + \beta - e_2^*] + v(\bar{\beta} | e_1) [\psi_2(\bar{e}_2) + \bar{\beta} - \bar{e}_2])] - (1 + \lambda)\psi_1(e_1)$$

Note that $g_2(e_1, \bar{e}_2) \geq 0 \Leftrightarrow 1 - \psi_2'(\bar{e}_2) \geq 0 \Leftrightarrow \bar{e}_2 \leq e_2^*$. Moreover

$$g_1(e_1, \bar{e}_2) = g_1(e_1, e_2^*) - (1 + \lambda) [e_2^* - \psi_2(e_2^*) - \bar{e}_2 + \psi_2(\bar{e}_2)] \times \frac{d[\pi(e_1)(1 - v(\beta | e_1))]}{de_1}$$

Note that $\psi_2'(e_2^*) = 1 \Leftrightarrow e_2^* - \psi_2(e_2^*) > \bar{e}_2 - \psi_2(\bar{e}_2) \forall \bar{e}_2 \neq e_2^*$. Moreover, $\frac{d\pi(e_1)(1 - v(\beta | e_1))}{de_1} > 0$ under Assumption 1.

Since $g(e_1, e_2^*)$ has its maximum in e_1^* , it comes immediately that for all \bar{e}_2 , there exists a value $\tilde{e}_1(\bar{e}_2) (< e_1^*)$ such that $e_1 \leq \tilde{e}_1(\bar{e}_2) \Leftrightarrow \frac{\partial g(e_1, \bar{e}_2)}{\partial e_1} \geq 0$. Finally, differentiating the expression $g_1(\tilde{e}_1(\bar{e}_2), \bar{e}_2) = 0$ with respect to \bar{e}_2 , we get that for all $\bar{e}_2 < e_2^*$, $\tilde{e}_1(\bar{e}_2)$ increases in \bar{e}_2 , and for all $\bar{e}_2 > e_2^*$, $\tilde{e}_1(\bar{e}_2)$ decreases in \bar{e}_2 .

Consider an indifference curve of level \bar{U}_{LL} such that $\bar{U}_{LL} \leq U_{LL}(e_1^*, e_2^*)$. Given the properties of indifference curves, there exist $e_1^a < e_1^*$ and $e_2^a > e_2^*$ such that $U_{LL}(e_1^a, e_2^a) = \bar{U}_{LL}$ and $U(e_1^*, e_2^a) = \bar{U}_{LL}$. We have

$$g_1(e_1^a, e_2^*)|_{\bar{U}_{LL}} = g_1(e_1, e_2^*)|_{e_1^a} + g_2(e_1^a, \bar{e}_2)|_{e_2^a} \frac{d\bar{e}_2}{de_1} > 0$$

$$g_1(e_1^*, e_2^a)|_{\bar{U}_{LL}} = g_1(e_1, e_2^a)|_{e_1^*} + g_2(e_1^*, \bar{e}_2)|_{e_2^a} \frac{d\bar{e}_2}{de_1} < 0.$$

Besides, for any effort e_1 and \bar{e}_2 such that $e_1 < e_1^a$ and $\bar{e}_2 < e_2^*$, $g_2(e_1, \bar{e}_2) > 0$ then $g(e_1, \bar{e}_2) < g(e_1, e_2^*) < g(e_1^a, e_2^*)$. Moreover, for all e_1 and \bar{e}_2 such that $e_1 > e_1^*$ and $\bar{e}_2 > e_2^a (> e_2^*)$,

$$g_1(e_1, e_2) + g_2(e_1, \bar{e}_2) \frac{d\bar{e}_2}{de_1} < 0.$$

Overall, conditional on leaving a rent equal to $\bar{U}_{LL} < U_{LL}(e_1^*, e_2^*)$, the optimal efforts are such that $\bar{e}_2 > e_2^*$ and $e_1 < e_1^*$. ■

Proof of Proposition 3:

(i) We can rewrite the expected utility of the firm as

$$U_{LL}(e_1, \bar{e}_2, k) = \pi(e_1)h(e_1, \bar{e}_2) - k - \psi_1(e_1) + \pi(e_1)v(\beta | e_1)\phi(\bar{e}_2) \equiv l(e_1, \bar{e}_2) - k.$$

Let us call \mathcal{P}_{LL}^k the optimization program of the regulator when the level of limited liability of the firm is k . Using the fact that $\underline{e}_2 = e_2^*$, it is simply

$$\begin{aligned} \mathcal{P}_{LL}^k : \max_{\{e_1, \bar{e}_2\}} & g(e_1, \bar{e}_2) - \lambda l(e_1, \bar{e}_2) + \lambda k \\ \text{s.t.} & l(e_1, \bar{e}_2) - k \geq 0 \\ & h(e_1, \bar{e}_2) \geq 0. \end{aligned}$$

The first constraint is (IR₁) and the second is (LL). Note that (MH) is reflected in $U_{LL}(e_1, \bar{e}_2, k)$. Then, the solution of this program satisfies (MH).

Suppose first that $k = 0$, and neglect the constraints of the program. Taking first-order conditions with respect to e_1 and \bar{e}_2 we have,

$$\begin{aligned} g_1(e_1, \bar{e}_2) - \lambda l_1(e_1, \bar{e}_2) &= 0 \\ g_2(e_1, \bar{e}_2) - \lambda l_2(e_1, \bar{e}_2) &= 0. \end{aligned}$$

Note that

$$\begin{aligned} g_1(e_1, \bar{e}_2) &= g_1(e_1, e_2^*) - (1 + \lambda)[e_2^* - \psi_2(e_2^*) - \bar{e}_2 + \psi_2(\bar{e}_2)] \\ &\quad \times \frac{d[\pi(e_1)(1 - \nu(\beta | e_1))]}{de_1} \end{aligned}$$

and $l_1(e_1, \bar{e}_2) = \frac{\partial U_{LL}}{\partial e_1} > 0$, then $e_1 < e_1^*$. Moreover $l_2(e_1, \bar{e}_2) = \frac{\partial U_{LL}}{\partial \bar{e}_2} < 0$, then $\bar{e}_2 < e_2^*$. Call $\hat{e}_1(0)$ and $\hat{e}_2(0)$ these two efforts. Suppose that $h(\hat{e}_1(0), \hat{e}_2(0)) \geq 0$ and $l(\hat{e}_1(0), \hat{e}_2(0)) \geq 0$, then $\hat{e}_1(0)$ and $\hat{e}_2(0)$ are the solutions of \mathcal{P}_{LL}^0 . The expected utility of the agent is $U_{LL}(\hat{e}_1(0), \hat{e}_2(0), 0) = l(\hat{e}_1(0), \hat{e}_2(0)) \geq 0$. Note also that given the properties of U_{LL} , we have $U_{LL}(\hat{e}_1(0), \hat{e}_2(0), 0) < U_{LL}(e_1^*, e_2^*, 0)$.

Suppose now $k > 0$. If the efforts are $\hat{e}_1(0)$ and $\hat{e}_2(0)$, the expected utility of the firm is $U_{LL}(\hat{e}_1(0), \hat{e}_2(0), k) = l(\hat{e}_1(0), \hat{e}_2(0)) - k$. The expected utility is decreasing in k , and there exists $\hat{k} (< |t_F^*|)$ such that for all $k < \hat{k}$, $U_{LL}(\hat{e}_1(0), \hat{e}_2(0), k) > 0$ and for all $k > \hat{k}$, $U_{LL}(\hat{e}_1(0), \hat{e}_2(0), k) < 0$. When $k < \hat{k}$, $\hat{e}_1(0)$ and $\hat{e}_2(0)$ are still the solutions of \mathcal{P}_{LL}^k . When $k > \hat{k}$, the regulator cannot satisfy (IR₁) by implementing the solution of the unconstrained problem. Then (IR₁) is binding. Consider $\bar{e}_2(e_1, k)$ such that $U_{LL}(e_1, \bar{e}_2(e_1, k), k) = 0$. We start with the following observations:

- (a) $\frac{\partial \bar{e}_2}{\partial e_1} > 0$ and $\frac{\partial^2 \bar{e}_2}{\partial e_1^2} > 0$ (see Appendix A2).
- (b) $\frac{\partial \bar{e}_2}{\partial k} \frac{\partial U_{LL}}{\partial \bar{e}_2} + \frac{\partial U_{LL}}{\partial k} = 0 \Rightarrow \frac{\partial \bar{e}_2}{\partial k} < 0$.
- (c) $\frac{\partial^2 \bar{e}_2}{\partial k \partial e_1} \frac{\partial U_{LL}}{\partial \bar{e}_2} + \frac{\partial \bar{e}_2}{\partial k} \frac{\partial^2 U_{LL}}{\partial \bar{e}_2 \partial e_1} = 0 \Rightarrow \frac{\partial^2 \bar{e}_2}{\partial k \partial e_1} < 0$.

- (d) $\frac{g_2}{\bar{e}_2}(e_1, \bar{e}_2) = -\pi(e_1)(1 - v(\beta | e_1))(1 + \lambda)[\psi'_2(\bar{e}_2) - 1] < 0$ for all $\bar{e}_2 > e_2^*$.
- (e) $\frac{g_{12}}{\bar{e}_2}(e_1, \bar{e}_2) = -\frac{\partial[\pi(e_1)(1-v(\beta|e_1))]}{\partial e_1}(1 + \lambda)[\psi'_2(\bar{e}_2) - 1] < 0$ for all $\bar{e}_2 > e_2^*$.
- (f) $g_{22}(e_1, \bar{e}_2) = -\pi(e_1)(1 - v(\beta | e_1))(1 + \lambda)\psi''_2(\bar{e}_2) < 0$.
- (g) Assumptions 1–3 and S large \Rightarrow
 $\frac{g_{11}(e_1, \bar{e}_2)}{de_1^2} = \frac{g_{11}(e_1, e_2^*) - (1 + \lambda)[e_2^* - \psi_2(e_2^*) - \bar{e}_2 + \psi_2(\bar{e}_2)] \times d^2[\pi(e_1)(1-v(\beta|e_1))]}{de_1^2} < 0$.

Let us neglect the constraint $h(e_1, \bar{e}_2) > 0$ for the time being. When $k \in (\hat{k}, |t_F^*|)$, (IR_1) is binding and the regulator chooses e_1 so as to maximize $g(e_1, \bar{e}_2(e_1, k))$. Taking the first-order condition with respect to e_1 , we have

$$g_1(e_1, \bar{e}_2(e_1, k)) + g_2(e_1, \bar{e}_2(e_1, k)) \frac{\partial \bar{e}_2}{\partial e_1}(e_1, k) = 0.$$

The solution of this equation is $\hat{e}_1(k)$. Differentiating with respect to k , we get

$$\begin{aligned} \frac{d\hat{e}_1}{dk} \left[g_{11} + 2g_{12} \frac{\partial \bar{e}_2}{\partial e_1} + g_{22} \left(\frac{\partial \bar{e}_2}{\partial e_1} \right)^2 + g_2 \frac{\partial^2 \bar{e}_2}{\partial e_1^2} \right] \\ + \frac{d\bar{e}_2}{dk} \left[g_{12} + g_{22} \frac{\partial \bar{e}_2}{\partial e_1} \right] + g_2 \frac{\partial^2 \bar{e}_2}{\partial k \partial e_1} = 0. \end{aligned}$$

Given (a)–(g) and using the fact that $\bar{e}_2 > e_2^*$ (see Appendix A2), we have $\frac{d\hat{e}_1}{dk} > 0$. Naturally, when $k \rightarrow |t_F^*|$, $\hat{e}_1(k) \rightarrow e_1^*$. Moreover, in equilibrium $\hat{e}_2(k) = \bar{e}_2(\hat{e}_1(k), k)$, then $\frac{d\hat{e}_2}{dk} = \frac{\partial \bar{e}_2}{\partial e_1} \frac{d\hat{e}_1}{dk} + \frac{\partial \bar{e}_2}{\partial k}$. If S is sufficiently large, $\frac{d\hat{e}_1}{dk}$ is small and $\frac{\partial \bar{e}_2}{\partial k} < 0$. When $k \rightarrow |t_F^*|$, then $\hat{e}_2(k) \rightarrow e_2^*$.³³ Last, since $\hat{e}_1(k) > \hat{e}_1(0)$ and $\hat{e}_2(k) < \hat{e}_2(0)$, we have $h(\hat{e}_1(k), \hat{e}_2(k)) \geq h(\hat{e}_1(0), \hat{e}_2(0)) \geq 0$, and (LL) is satisfied.

Overall, if the solution of the unconstrained problem satisfies (IR_1) and (LL) when $k = 0$, then it is the solution of \mathcal{P}_{LL}^k for all $k < \hat{k}$. When $k \in [\hat{k}, |t_F^*|]$, (IR_1) is binding and (LL) is always satisfied. Naturally, if the solution of the unconstrained problem does not satisfy either (IR_1) or (LL) when $k = 0$, the solution is constrained. Given that $h(e_1, \bar{e}_2)$ and U_{LL} have the same properties (increasing in e_1 and decreasing in e_2), the solution obtained whenever either (IR_1) or (LL) is binding is qualitatively similar to the solution obtained above (first-stage effort increasing in k and second-stage effort decreasing in k).

³³ $\hat{e}_2(k)$ might increase in k on some range if S is not sufficiently large. However, it eventually decreases to e_2^* .

- (ii) It is a direct consequence of (i). In particular, if the solution of the unconstrained problem satisfies the constraints in $k = 0$, then U_{LL} is positive and decreasing when $k < \hat{k}$ and equal to 0 for all $k \in [\hat{k}, t_F^*]$. ■

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