The Journal of Risk and Uncertainty, 22:2; 141–164, 2001

Rush and Procrastination Under Hyperbolic Discounting and Interdependent Activities*

ISABELLE BROCAS brocas@ulb.ac.be
ECARES and CBDR, Université Libre de Bruxelles, 1050-Brussels, Belgium

JUAN D. CARRILLO carrillo@ulb.ac.be
ECARES and CEPR, Université Libre de Bruxelles, 1050-Brussels, Belgium

Abstract

We analyze the decision of individuals with time-inconsistent preferences to invest in projects yielding either current costs and future benefits or current benefits and future costs. We show that competition between agents for the same project mitigates the tendency to procrastinate on the first type of activities (i.e. to undertake them "too late") and to rush on the second one (i.e. to undertake them "too early"). Competition can therefore increase the expected welfare of each individual. On the contrary, complementarity of projects exacerbates the tendency to rush and to procrastinate and therefore it can decrease the expected welfare of each individual.

Keywords: time inconsistency, rush, procrastination, competition, cooperation

JEL Classification: A12, D81, D90, J22.

There is an innate human tendency both to delay unpleasant tasks and to succumb to temptations by rushing into pleasant activities. On the one hand, procrastination occurs even under the anticipation that, sooner or later, the referee report has to be completed, the family dinner invitation accepted, and the bedroom shelf fixed. On the other hand, "freezing" (in its literal sense) the credit card and writing a contract that self-forbids the entrance in casinos is sometimes the only way to avoid an impulsive behavior that has pernicious long-run consequences.

The literature on behavioral economics has shown that the tendency to undertake activities "too late/too infrequently" (procrastination) or "too early/too often" (rush) may result from the combination of dynamically inconsistent preferences and a temporal gap between the costs and benefits associated to those actions. The idea that preferences are dynamically inconsistent or, more precisely, that the individual period-to-period discount rate falls monotonically is becoming more and more accepted in Psychology and in Economics. It has received the support in a large series of experiments in both fields.

*We thank Andrea Prat, Stefano Della Vigna and an anonymous referee for helpful comments. Financial support from the European Commission TMR Marie Curie Fellowship BICT 972479 (Isabelle Brocas) and TMR network FMRX CT-96-0028 (Juan D. Carrillo) is gratefully acknowledged.

From a theoretical perspective, Strotz (1956) and Phelps and Pollak (1968) are the first studies in which individual and social dynamically inconsistent preferences were analyzed, respectively.[1] The purpose of this work is to extend in several directions the previous analyses related to the effects of time inconsistent preferences (hyperbolic discount rates) on the behavior of individuals. We consider a population of time inconsistent agents who can undertake one or several irreversible activities. The horizon is infinite (or stochastic), so as long as agents choose not to engage in the activity, there is always scope for undertaking it in the future. Two different scenarios are studied. In the first one, the activities require a current cost but provide a future benefit. In the second one, the activities yield a current benefit at the expense of a delayed cost. For both types of scenarios, we analyze the decision of agents who can undertake several independent activities (simultaneously or sequentially), complementary activities, or competing activities. Our main contribution is to show that, in some environments, competition between agents may mitigate the usual inefficiencies due to time inconsistency (rush and procrastination) and therefore be welfare enhancing. By contrast, in complementary activities the inefficiencies are exacerbated, so that cooperation can be welfare damaging. Naturally, this contrasts with the standard theory with time consistent (exponential) discounting, in which agents can only benefit from complementarity and be individually impaired by competition.

The following example will help to illustrate in detail each of our findings in the procrastination case (Section 1). Consider the decision of a researcher (he) to start new, difficult projects instead of pursuing simple routine activities. Costs on the ambitious tasks (time, effort, etc.) come on average earlier than benefits (career concerns, self-esteem, etc.). Therefore, according to the theory (Akerlof (1991), O'Donoghue and Rabin (1999a, 1999b) and others), the time inconsistent individual is likely to procrastinate on them. In our example, the researcher will for instance work only on a fraction of projects at each period and delay the rest for the future. Our first result says that the welfare of the researcher will not be reduced if he is constrained to undertake projects sequentially (Proposition 1(ii)) or if he competes against other researchers in a winner-takes-all project completion game (Proposition 2(i)). The two situations are very different but the idea behind the results is indeed quite similar. In both cases, there is an extra cost of delaying the completion of a given project: under sequentiality, it postpones the possibility of undertaking all future ones, and under project competition it increases the probability of not being the first one to undertake it. The researcher internalizes this cost and reacts by reducing the amount of inefficient procrastination. As a result, he ends up completing projects earlier (on average) than without the constraints and, at the same time, keeps the same expected benefit. Therefore, apparent work overload (i.e. the tendency to accept too many tasks when these cannot be completed immediately) may just be a commitment device by a researcher to avoid postponing indefinitely valuable projects that require time and effort. Similarly, inducing competition between scientists (for example through competition for research grants) reduces procrastination and results in earlier discoveries without decreasing the welfare of each particular individual. It is interesting to notice that, in the competition case, there is a coordination game in which each researcher is willing to delay the completion of the project only if the competitor also does.

This immediately implies the existence of multiple equilibria characterized by the degree of procrastination of both individuals. We then study the effects of project complementarity by assuming that researchers can engage in a joint activity. More precisely, as in any Research Joint Venture or co-authored work, each individual is responsible for part of the project and a joint benefit is obtained whenever both of them have completed their job. Interestingly, complementarity exacerbates the incentives of researchers to procrastinate. Fulfilling their own duties is not anymore sufficient to enjoy the benefits. Each researcher is forced to rely on his teammate's willingness to do his part of the job, but realizes the natural tendency to procrastinate of his partner. As a result, in some cases each researcher will optimally want to complete his assignment at each date with a positive but smaller probability than his teammate. This situation is characterized by the greatest possible inefficiency: in the unique symmetric equilibrium, no researcher ever fulfills his job even though the joint project has positive expected value for both of them (Proposition 2(ii)). Note that this occurs even if effort is observable, which rules out the usual free-riding problem.

The second part of the paper (Section 2) deals with situations in which undertaking an activity yields an immediate benefit and a delayed and uncertain cost. One can think of politicians willing to undertake a public project (library, hospital, etc). The investment yields current benefits in terms of prestige and chances of reelection, and it is financed over time (so, at least in part, by future politicians in office). Again according to the usual theory (O'Donoghue and Rabin (1999a), Brocas and Carrillo (1999, 2000) and others), politicians will have a tendency to rush, which in this case means accepting projects with an excessively high cost. We assume that the cost of the project varies stochastically and that the politician learns at each date the realization of the cost for the project in the current period. In this setting, we show that two equilibria may coexit: One in which the politician waits until the cost is sufficiently low and one in which he rushes and accepts to finance a project even if it has a negative Net Present Value (NPV) (Proposition 3(i)). Naturally, the expected intertemporal payoff of the politician is higher in the first equilibrium than in the second one. However, he is trapped by the expectation of his future behavior. Anticipating a future inefficient behavior (rush) "forces" him to rush in the current project, so as to at least reap the overweighed current benefits of investing in it. Similarly, anticipating patience and no rush in the future incites him to be patient and finance the current project only if its NPV is positive and sufficiently high. We then study the case of two neighboring jurisdictions competing for one project. This situation arises when duplication of projects is inefficient (e.g. a football stadium) and therefore only the first politician to make an "acceptable" proposal will be entitled to finance the project in his own jurisdiction. If in the single-project, single-politician equilibrium there is no rush, then project competition between jurisdictions can only decrease the individual welfare of the politician. More interestingly, if the equilibrium in the single-project, single-politician case implies rush, then the welfare of the politician will be strictly increased by allowing competition for projects (Proposition 4(i)). The key for the result relies on the fact that politicians anticipate that rushing is detrimental. Yet, given their intrapersonal conflict of preferences, it is a commitment device against future decisions even more inefficient from the current perspective (see the previous result).

Under competition, only the first proposal can be financed, so commitment against future undesirable behavior can be achieved at no cost: each politician will try and use the intrapersonal conflict of the rival to his advantage by "letting the opponent rush". In other words, a politician may accept to finance a current excessively costly project if he anticipates that otherwise he will be tempted to finance an equally costly project in the future. But, if the neighboring jurisdiction incurs itself in this wasteful expense, then the commitment not to finance the project in the future is achieved without any current disbursement. Overall, competition strictly increases individual welfare. Last, we analyze complementary projects. Some investments in one jurisdiction may generate positive spillovers in the surrounding districts if the latter also finance their own public projects. For instance, the benefits of developing a railroad node or a TV station will be greater if these services already exist in the neighboring jurisdiction. Complementarity increases the net benefits of financing the public project, and as a result it also exacerbates the tendency of politicians to rush (Proposition 4(ii)). Hence, project complementarity may reduce the welfare of individuals. Put it differently, a politician in search of prestige may not only incur in unnecessary expenses in his own jurisdiction, he will also encourage this behavior in the leaders of the surrounding districts. The relevance and practical implications of these results for other economic issues such as promotions, job search, R&D cooperation, individual temptations, etc. are discussed at the end of the paper. It is important to notice that the findings emphasized in Propositions 2 and 4 crucially rely on the interaction between interpersonal and intrapersonal conflicts. Without time inconsistent preferences, competition and complementarity cannot reduce individual welfare and increase individual welfare, respectively.

Before presenting the model, we would like to give a brief overview of previous contributions to the literature on decision making under time inconsistency. Akerlof (1991) is the first applied paper in the field. He shows that time inconsistency induces procrastination and severe welfare losses if the agent (wrongly) thinks that only current preferences are time inconsistent. O'Donoghue and Rabin (1999a) compare the decision of agents who correctly anticipate that the time inconsistency problem is present at every period (sophisticated) with that of individuals who are not aware of their future self-control problem (naïve). In a single task case and a deterministic setting, they demonstrate that sophisticated agents are less likely to procrastinate but more likely to rush than their naïve peers. Brocas and Carrillo (1999) introduce uncertainty and exogenous revelation of information in a model where agents are sophisticated and benefits of the activity come earlier than costs. As in O'Donoghue and Rabin (1999a), agents can rush (i.e invest with negative NPV). Their contribution is to show that the likelihood of incurring in severe welfare losses depends (non monotonically) on the flow of information transmitted between periods: rush may occur under meager per-period information transmission but never if the flow of information is either zero or substantial. Last, O'Donoghue and Rabin (1999b) generalize their previous work and show that welfare losses incurred by naïve agents in situations where costs come earlier than benefits persist if agents are partially aware of their self-control problem, and if they face more options than the standard binary choice problem. Within this literature, the present work then replicates previous findings related to the tendency of sophisticated agents to rush and procrastinate.

However, its original contribution is to analyze the effects of sequential task completion, interpersonal competition and interpersonal complementarity on the decision of time inconsistent agents. To the best of our knowledge, this is the first attempt to combine an intrapersonal together with an interpersonal conflict of preferences.

The paper is organized as follows. Section 1 investigates the decision of researchers to start scientific projects in which costs are salient. Section 2 studies the decision of politicians to finance public projects for which private benefits are salient. In both cases we provide results for the cases of independent and interpendent projects. Section 3 discusses some other applications of our theory and Section 4 concludes. Proofs of the propositions are relegated to the appendix.

1. A simple model of procrastination

In this section, we study the decision of researchers to pursue projects that require *current net costs* in terms of intellectual effort or time but provides *long-term benefits*, such as better career prospects or a better self-perception. Each agent has time inconsistent preferences in the sense of Strotz (1956) so that short term events are discounted relatively more heavily than long term events. For each researcher (agent), we call "self-t" his incarnation at date t. For analytical tractability, we use the quasi-hyperbolic discounting introduced by Phelps and Pollak (1968). According to their modeling, period $t + s$ is, from the perspective of self-t, discounted at a rate $\beta\delta^s$ with $\delta < 1$ and $\beta \in (0, 1)$.[2] In order to keep notations as simple as possible, we assume that undertaking the research project in period t has an immediate cost of effort e at date t and a delayed positive payoff π at date $t + 1$.[3] We focus on the case of a research project. Other economic applications of our theory such as job search, R&D innovations and promotions within firms are discussed in Section 3.

According to our payoff structure, self-t prefers to do the activity at date t rather than not doing it ever if and only if:

$$-e + \beta\delta\pi > 0 \Leftrightarrow e < \bar{e} \equiv \beta\delta\pi \tag{1}$$

Similarly, self-t prefers to do the activity at date $t + 1$ rather than at date t if and only if:

$$-e + \beta\delta\pi < -\beta\delta e + \beta\delta^2\pi \Leftrightarrow e > \underline{e} \equiv \beta\delta\pi\frac{1-\delta}{1-\beta\delta} \tag{2}$$

We will assume that both (1) and (2) hold simultaneously. This is possible only because of the dynamic inconsistent nature of preferences ($\beta < 1$).

Assumption 1

$$e \in (\underline{e}, \bar{e}). \tag{A1}$$

We will investigate two scenarios. In the first one, researchers undertake independent projects, and therefore the payoff obtained by each individual is not affected by the decision of the others. In the second one, researchers either compete in a race where all the benefits are captured by the winner (negative externality) or they cooperate in a joint project as coauthors (positive spillovers). Naturally, in the competition and cooperation cases, the payoff obtained by each researcher is affected by the decision of the other agents. Our first result is a characterization of the behavior of agents who pursue independent projects.

Proposition 1 *(Procrastination under independent projects). Suppose that researchers can undertake n independent and identical projects:*

(i) *In the unique symmetric Subgame Perfect Equilibrium, researchers complete at each date a fraction* $\lambda^* \in (0, 1)$ *of projects.*
(ii) *If the researcher cannot perform more than one project per period, there is no loss in welfare as long as* $n \leq 1/\lambda^* + 1$.

Proof. See Appendix 1. □

First, under time inconsistent preferences, it may occur that the researcher wants the projects to be completed but, at the same time, prefers to delegate their realization to future incarnations. Therefore, given the natural tendency to procrastinate, our theory predicts that the agent will complete only a fraction of projects at each period even in the absence of increasing marginal costs (instead of all or none at the beginning, as in the traditional theory). From the definition of λ^*, note that procrastination increases as the intrapersonal conflict of preferences becomes more acute, the cost of effort becomes greater and the benefit of the project becomes smaller $((\partial\lambda^*(\cdot)/\partial\beta) > 0, (\partial\lambda^*(\cdot)/\partial e) < 0, (\partial\lambda^*(\cdot)/\partial\pi) > 0)$. More interestingly, the project is more likely to be delayed, when, keeping the current net benefit $(\beta\delta\pi - e)$ constant, the stakes e and π are increased $(\beta\delta(\partial\lambda^*(\cdot)/\partial\pi) + (\partial\lambda^*(\cdot)/\partial e) < 0)$. Since present costs are overweighed relative to future returns, an increase in the stakes raises the net payoff of delegating the project to future incarnations. Therefore, contrary to common wisdom, procrastination is more likely to occur the more valuable the project is. Overall, one cannot neglect the problem of self-control based on the fact that it mainly affects decision making in situations of limited importance. Note that a similar result was already obtained by O'Donoghue and Rabin (1999b): in that model, it is also true that the inefficiency is more likely to be important if the task is highly valuable. However, the reason is the agents' (total or partial) naïvete.

Second, when the researcher is forced to undertake projects sequentially (for example because the results of the first project are used in the second one), completing a given project has the added implicit benefit of making possible the completion of future ones. In other words, there is an option value of undertaking the current research activity. This option value is an increasing function of the number of remaining projects, and it affects negatively the researcher's incentives to procrastinate. That is, the higher the number of future projects to be done is, the higher the benefit of completing the current one, and so the higher the equilibrium probability of effectively undertaking it. The interesting (although at first counterintuitive) property is that if the procrastination problem is acute, then the researcher may not suffer any welfare loss due to the impossibility of undertaking projects simultaneously. One could think that introducing a constraint on the timing of project completion should always decrease the agent's welfare (as it is the case under time consistent preferences). However, as described above, imposing sequentiality only decreases the incentives to procrastinate by endogenously increasing the cost of delaying projects. The researcher then reacts by undertaking the earliest projects with a higher probability, and keeps the same expected utility. Naturally, as the procrastination problem becomes less important (β gets larger), then forcing the agent to a sequential project completion is more likely to reduce welfare.[4] Overall, our result suggests that apparent work overload (i.e. the tendency of researchers to commit to too many projects even if it is not feasible to complete all of them immediately) may not be welfare damaging. It can simply be a commitment device to overcome a natural tendency to procrastinate.

In the remaining of the section, we consider the case in which projects are interdependent, so that the payoff of each researcher is affected by the decision of his peers. First, we analyze *competition* between researchers. Two individuals may independently embark on the same research project, and then the full credit goes to the first one to complete it. To keep the analysis simple and the notations as close as possible to the independent project situation, we suppose that each researcher can undertake his project and reap the entire benefit π if and only if his rival has not undertaken his own one in a previous period. Besides, if both agents complete their projects simultaneously, they both get the benefit π.[5] Second, we study the case of *complementary* projects. Two coauthors may start a joint research activity. The work needs the input of both researchers and it is only when the entire project is completed that the two individuals enjoy a (shared) benefit. Once again, other possible examples of competing and complementary activities are mentioned in Section 3. In this setup, we can state our next result.

Proposition 2 *(Procrastination under interdependent projects). There exist two cut-off efforts e^* and e^{**} that determine the stable, symmetric equilibria of the game.*

(i) *When agents are in competition, they both complete the project in the first period with probability 1 if $e \in [\underline{e}, e^*)$ and at each period with probability $p \in \{\mu, 1\}$ (where $\mu > \lambda^*$) if $e \in [e^*, \bar{e}]$.*
(ii) *When agents pursue complementary projects then, as long as nobody has undertaken his own job, agents complete them with probability γ $(<\lambda^*)$ at each period if $e \in [\underline{e}, e^{**})$ and with probability 0 if $e \in [e^{**}, \bar{e}]$. Moreover, for some pairs (β, δ) then $e^{**} = \underline{e}$, i.e. never completing the task is the only symmetric equilibrium.*

Proof. See Appendix 2. □

When researchers compete for the same project, their tendency to procrastinate is affected not only by the anticipation of future behavior, but also by the tendency to procrastinate of their rival. Given that projects are valuable, if the cost of effort is sufficiently low ($e < e^*$) each agent prefers to undertake it immediately, fearing his opponent's behavior and the possibility of not reaping any benefit due to procrastination. More surprisingly, for intermediate values of effort ($e \in (e^*, \bar{e})$) time inconsistency introduces an interpersonal coordination problem. Researchers are willing to procrastinate and delay the completion of the project. However, they will effectively do it only if they anticipate that their rival will also procrastinate with some probability. This gives rise to a multiplicity of equilibria, each of them characterized by the probability of the project being realized by each individual at each date. By construction, all (symmetric) equilibria yield the same expected payoff to each agent ($\beta\delta\pi - e$). Overall, introducing competition between researchers endogenously increases the cost of procrastination. Individuals then react by undertaking their project sooner (on average) than under independent tasks, without incurring in any welfare loss. Confronting researchers is then an efficient and costless way to overcome their natural tendency to delay activities with immediate costs. Obviously, if early completion is beneficial for a third person (e.g. the society may benefit from frequent research discoveries) then introducing competition is a Pareto improvement for the economy. Last, note the similarities between the results for sequential and competing projects: in both cases the prospects of a possible welfare loss decreases the incentives to procrastinate without affecting the final utility of the individuals.

The analysis is different when two coauthors cooperate in a joint research project. If one researcher has already fulfilled his part of the job, then the team mate completes his own duties with probability λ^* at each period. This is simply because the agent faces at that point the same decision problem as under independent projects (see Proposition 1(i)). However, complementarity of projects increases the overall willingness of agents to delay the realization of their task: exerting effort is not anymore sufficient to enjoy the benefit at the following period, so there is an extra incentive to procrastinate. As a result, the equilibrium probability of completing the task is at most γ ($<\lambda^*$). The most striking feature of the equilibrium under cooperation is the welfare loss resulting from the fact that two coauthors may *never* complete a project yielding net profits to both of them. This may happen for two different reasons. First, a trivial one. If e is close enough to $\bar{e}$, then the project is valuable only if both researchers undertake their part of the job in the same period. However, each of them has incentives to deviate and wait until the other has incurred the cost before exerting the effort himself. This free-riding problem results in an inefficiency because no individual is ever willing to take the first step.[6] Second and more surprisingly, if $\delta < 1/(1 + \sqrt{1-\beta})$, then the free-riding problem arises *for all e* satisfying (A1), so that the unique symmetric equilibrium is to never do the project. Basically, for any given probability that the team mate undertakes the project, each researcher is willing to complete his own job with a positive, but always smaller probability than his colleague. Again, this leads to an inefficient, unique symmetric equilibrium in which, because of a coordination problem, projects with a strictly positive (and even deterministic) net value remain unfulfilled forever. Hence, contrary to common wisdom, it can be inefficient for time inconsistent individuals to engage in cooperative activities. Notice that as long as δ is not too close to 1, this inefficiency arises even if the intrapersonal conflict is very small (i.e. even if β is close to 1).[7]

2. A simple model of rush

We now analyze a situation in which time inconsistent political leaders can approve projects that are partially irreversible. These projects are financed over long periods so they entail a *delayed cost*. At the same time, they procure a private *current benefit* (in terms for example of prestige). Formally, and by symmetry with the previous section, we assume that if self-t undertakes the activity, he enjoys a benefit x at date t and pays a cost c_t at date $t+1$. Besides the change in the temporal sequence between costs and benefits, there is a second difference with the setup of the previous section: we will assume that the cost c_t incurred at $t+1$ if the project is approved at t is not deterministic but rather a random variable drawn form a common knowledge distribution with c.d.f. $F(c)$. At each period t and before making his decision, the politician learns the realization c_t of the cost to be paid at $t+1$ in case of approving the current project. A stochastic evolution of costs simply reflects the fact that projects valuable at some date can become obsolete in the future or that there is gradual learning about the value of the potential investments. This evolution of costs is formalized as follows.[8]

Assumption 2

$$c_t \text{ i.i.d. } \mathcal{N}(m, 1). \tag{A2}$$

We will focus on Markov Perfect Equilibria (MPE) for which the realization of the cost c is the state variable. Our first preliminary result says the following.

Lemma 1. *An MPE is characterized by a cutoff value c^* such that self-t finances the project at date t if and only if $c_t \leq c^*$. Such an MPE will be denoted by $[c^*]$.*

Proof. Assume that at date t the project has still not been financed. Besides, self-t anticipates that, from next period on, all future selves τ $(\geq t+1)$ will choose to finance it if and only if $c_\tau \leq c^*$. For a current realization c_t, self-t prefers to finance the current project if and only if:

$$x - \beta\delta c_t \geq \beta\delta F(c^*)(x - \delta E[c|c \leq c^*]) + \beta\delta^2(1 - F(c^*))F(c^*) \times (x - \delta E[c|c \leq c^*]) + \cdots = \beta\delta F(c^*)\frac{x - \delta E[c|c \leq c^*]}{1 - \delta(1 - F(c^*))}$$

where the left-hand-side (l.h.s.) represents the net benefit of the current project and the right-hand-side (r.h.s.) is the expected benefit from the current perspective of not financing the current project given that, in this case, it will be financed whenever its costs becomes smaller than c^*. Note that the l.h.s. of the inequality is decreasing in c_t. Therefore, an equilibrium strategy must specify a cutoff below which the agent undertakes the project. Rearranging terms we get:

$$x[(1-\delta) - \delta F(c^*)(1-\beta)] \geq \beta\delta[c_t(1-\delta) + \delta F(c^*)(c_t - E[c|c \leq c^*])] \tag{3}$$

Overall, from (3) we note that if an MPE exists, it must satisfy:

$$x[(1-\delta)+\delta F(c^*)(1-\beta)]=\beta\delta[c^*(1-\delta)+\delta F(c^*)(c^*-E[c|c\leq c^*])]$$

where c^* is the cutoff below which the politician finances the current project. □

Denote by $\hat{c}$ the value of the cost such that the project has zero NPV from the current perspective. Formally:

$$x=\beta\delta\hat{c} \tag{4}$$

In this framework, we will say that an MPE $[c^*]$ implies "rush" if $c^* > \hat{c}$, that is if the politician may choose to finance a project with negative, NPV ($c \in (\hat{c}, c^*)$). Naturally, an MPE $[c^*]$ never implies rush if and only if $c^* < \hat{c}$.

As in the previous section, we first consider the case in which projects in different jurisdictions are independent. Then, each politician chooses the date in which he finances projects in his own jurisdiction independently of the behavior of his neighboring leaders. In a second step, we investigate scenarios in which decisions in a given jurisdiction affect choices and payoffs in the surrounding ones. When projects are independent, we get the following result.

Proposition 3 *(Rush under independent projects). Suppose that politicians can finance at most two independent and identical projects:*

(i) *For the realization of each project there exist at most two stable MPEs $[c_1^*]$ and $[c_2^*]$. Moreover, the agent may rush in one MPE but not in the other ($c_2^* > \hat{c} > c_1^*$). Last, if $c_2^* > c_1^*$, the ex ante expected welfare for all selves is smaller is $[c_2^*]$ than in $[c_1^*]$.*

(ii) *When projects have to be undertaken sequentially, there exist at most two stable MPEs $[\tilde{c}_1]$ and $[\tilde{c}_2]$ for completing the first one. Moreover, $\tilde{c}_1 > c_1^*$ if $c_1^* < \hat{c}$ and $\tilde{c} < c_1^*$ if $c_1^* > \hat{c}$ (and similarly for $\tilde{c}_2$).*

Proof. See Appendix 3. □

First, as mentioned in the introduction, the possibility that a time inconsistent individual incurs in expenses with negative NPV (what we identify by "rush" or "haste") is not a new result. It has already been emphasized by O'Donoghue and Rabin (1999a) when costs and benefits are deterministic but non-stationary and by Carrillo and Brocas (1999) for stochastic payoffs. The first new result in Proposition 3 is the existence of multiple equilibria where the politician may and may not rush.[9] As in the context of procrastination, agents behave strategically against their future selves. However, unlike in the previous setup, when benefits come earlier than costs this intrapersonal conflict translates into an *intrapersonal coordination problem*. Politicians will then refrain themselves from financing projects with relatively bad prospects or not depending on the "degree of trust on their own future behavior". It is interesting to notice that a political leader may greatly benefit from building a reputation for being *patient* not only vis-à-vis of the electorate but even vis-à-vis of himself.[10] More importantly, when politicians are restrained to finance projects sequentially, their decision to embark on one of them depends on the anticipation of their future behavior. Naturally, a politician who has to decide whether to complete the first project in the current period is better-off if his future selves plan not to rush when completing the second one. The interesting result is that if the political leader anticipates rush in the second project, then he is less prone to rush in the first one with the sequentiality restriction than without it. Patience is, in that case, the best commitment device to delay as much as possible that future action which is recognized as being inefficient from the current viewpoint. Conversely, anticipating no rush in the second task makes the agent more willing to rush on the first one. We thus get a stronger result than in Proposition 1(ii): when $c^* > \hat{c}$, imposing a sequential completion of projects is a disciplining device for politicians which improves the welfare not only of the society but even of the political leader himself.

As in the previous section, the second objective is to investigate a scenario in which the payoffs of individuals are interdependent. First, we are concerned with situations in which politicians in neighboring jurisdictions implicitly *compete* for public projects. This might be the case when we consider the construction of a sports center or a concert hall. If duplication of these public goods is inefficient and costs are stochastic, the two politicians will engage in a race in which only the first one to make a "sensible" proposal may be able to undertake the project in his own jurisdiction. Second, we study cases in which public projects are *complements* and generate positive spillovers. For example, the construction of a new railroad or communication center in a given jurisdiction is more valuable the more developed these same infrastructures are in the neighboring districts. In this setting, we can state the following result.

Proposition 4 *(Rush under interdependent projects).*

(i) *When agents pursue competing projects, they are less likely both to rush and to make high profits.*
(ii) *When agents pursue complementary projects and the average cost is sufficiently high, they are more likely to rush.*

Proof. See Appendix 4. □

The effect of project competition on the behavior of politicians is twofold. First and not surprisingly, it lowers the maximum expected payoff. Individuals are concerned about the possibility of being leapfrogged by their rival, so they are willing to sacrifice some of the benefits of waiting. As in the traditional literature on investment under uncertainty, competition decreases the option value of waiting. Second and more interestingly, competition can mitigate the inefficiency due to time inconsistency, and therefore end up being welfare improving. For instance, in a situation where politicians have a tendency to rush and finance worthless projects, allowing competition decreases the incentives of individuals to undertake them. The key idea is that the political leaders are aware of the inefficiency of rushing, but they use it as a commitment device against a future behavior even more inefficient from the current perspective. In this setting, competition decreases the pressure to finance a project with expected losses: the rival may finance it in his own jurisdiction, in which case the commitment against future negative payoffs is achieved at no cost. In other words, by introducing competition, politicians do not become more patient, but they try to let the rival rush. Notice that our model provides clear predictions about the situations in which competition increases and decreases welfare, respectively. If, under independent projects, politicians never finance projects with negative NPV (i.e. if $c^* < \hat{c}$) then competition decreases welfare (the new cutoff becomes $c^{**} > c^*$).[11] By contrast, if rush under independent projects is possible ($c^* > \hat{c}$), the alleviation of the self-control problem offsets the standard decrease in the option value of waiting and competition becomes beneficial (i.e. $c^{**} < c^*$). Last, note that just like in the procrastination case, the results are similar when projects are independent but have to be undertaken sequentially and when projects are in competition.

By contrast, complementarity may exacerbate the incentives to rush. Indeed, the presence of spillovers increases the benefits of each public project. This has two effects going in opposite directions. First, the overall payoff of each politician increases if the cost remains unchanged. Then, complementarity enlarges the set of costs for which undertaking the project is efficient. Second, the politician being second to finance the project in his district enjoys the positive externality immediately, which is therefore overweighed. As a result, he may be more likely to invest with a high cost and take a more inefficient decision than in the absence of spillovers. Overall, if the increase in the benefit is smaller than the increase in the cost under which the decision is taken, then the second effect dominates. This is more likely to occur when the cost is high on average. Indeed, given the current realization of the cost, the politician being second to undertake the project has more incentives to finance it today if he anticipates that the future cost will be high on average. Combining this with the intrapersonal conflict of preferences, he is more likely to rush when the benefit is increased by a positive externality. This in turn also increases the incentives to invest of his colleague. Overall, the desire to reap current private benefits may push politicians to build impressive public buildings that quickly become obsolete. Moreover, complementarity has a snowball effect: unreasonable expenses in public projects are more likely to be imitated in neighboring districts whenever they generate spillovers. For example, several local politicians may decide to promote the construction of an excessively costly road network simply because the surrounding localities are already engaged in this project. The overall conclusion is the same as in the previous section: time inconsistency may twist the standard results about the effects of sequential completion of projects, competition and complementarity on the decision making and welfare of individuals.

3. Other applications

3.1. Some implications of and solutions to procrastination

Many economic situations are characterized by the existence of an immediate cost and a delayed benefit. In this section we provide a series of prescriptions for some of these cases.

Promotions. An important issue in the Theory of Organizations is to understand how managers may provide optimal incentives to their employees. The presence of asymmetric information and moral hazard concerns has been identified as a source of conflict between the two parties (managers and employees) that can be handled with the use of incentive contracts. In this context, promotions can be an effective reward to increase the performance of agents. According to the results of Section 1, competition for a promotion can be extremely beneficial: it will reduce the agents' tendency to procrastinate on their productive tasks. This will increase the utility of the manager without implying any welfare loss to agents. By contrast, setting a promotion scheme contingent on the realization of a joint activity of several agents is harmful as it will only magnify their willingness to delay unpleasant tasks.

Cooperation in R&D. Cooperation has been extensively analyzed in the R&D literature. The main drawback of allowing research laboratories to engage in Research Joint Ventures is the free riding problem when efforts are not observable. In our framework there is a qualitatively similar inefficiency although more extreme, and even under perfect observability of effort and complete information. Indeed, we show that R&D cooperation exacerbates the tendency to procrastinate, since agents are all the more reluctant to exert the current costly effort as the expected delay to obtain the benefit is high. As a result, valuable joint projects may *never* be started. Hence, if a regulator wants to foster R&D innovations, she has to realize that cooperation may be harmful not only for consumers, but even for the firms themselves. Naturally, there are some possible ways of keeping the benefits of cooperation and at the same time avoid some of the costs, like intermediary remuneration to each laboratory for partial fulfillment of the project. Our point is just that under hyperbolic discounting cooperation per se will not necessarily increase the speed of innovations.

Job search. An unemployed agent will decide whether to search for a job depending on the size of the expected future benefits of being employed relative to the current search costs. Under time inconsistent preferences, agents procrastinate in their search activity. As a result, they remain on average out of job during an inefficiently long period of time. When there are few job openings, workers fiercely compete for them. What our analysis suggests is that in periods of job scarcity the incentives of agents to delay or decrease the intensity of their job search are weak. Hence, positions will be fulfilled on average sooner without necessarily affecting negatively the agents' utility.[12]

3.2. Some implications of and solutions to rush

We now consider activities characterized by an immediate benefit and a delayed cost. Recall that, in those situations, individuals may invest with a negative payoff. In addition, we have evidenced the presence of multiple equilibria which reflects that the tendency to rush depends on the degree of trust on future behaviors.

Personal temptations. It is widely argued that human beings have an innate tendency to succumb to all sorts of temptations. From a general perspective, a temptation can be defined as the desire to undertake activities that provide an immediate "mixed feeling". A clear illustration is impulse buying, which roughly corresponds to a willingness to acquire goods anticipating regret once the purchase is realized (see e.g. Rook, 1987). Other activities such as gambling or extramarital relationships provide the same kind of feeling. Some explanations relying mainly on bounded rationality (the existence of temporary and unanticipated urges) have been provided. However, these arguments do not incorporate the idea that the temporal gap between costs and benefits of the activity is a key factor that determines the likelihood of exhibiting impulsiveness (e.g. buying on impulse is more frequent for credit card payments). By contrast, dynamically inconsistent preferences brings a clear answer to this behavior. In the absence of credit facilities, agents will never acquire useless goods that they can barely pay. However, if they are allowed to postpone payments, a purchase with regret (even at the current date) may occur. According to the results of our model, building some self-reputation can be the only allied of the individual in order to mitigate his tendency to act impulsively. Only if he anticipates that not falling in the temptation currently is an indicator that he will refrain from succumbing in the future, will the agent be able to avoid satiating immediate pleasure with long-run harmful consequences.[13]

Staying on the job. Our model can be reinterpreted in terms of the willingness of agents to change their job in uncertain environments. It is usually difficult for an agent to search for a different occupation when he is already employed. His main options are therefore either to keep his current position or to resign and look for a better one. In traditional job search theory, the opportunity cost of not searching for another occupation can never exceed the value of the current job otherwise the agent would strictly prefer to quit. This paper claims that too much conservatism in the decision to remain in the current activity may not be due to high risk aversion but rather to time inconsistent preferences. It is interesting to note that agents may procrastinate in their job search (see section 3.1) and, at the same time, exhibit an excessively high willingness to keep a position once they accept it. Once again, under job scarcity and tight job competition, agents are less likely to adopt a conservative strategy. Surprisingly, this can enhance their welfare.

4. Concluding remarks

Accounting for time inconsistent preferences may change our interpretation of individual and collective behavior in economic activities as diverse as the financing of public projects, research discoveries, job search, or consumption decisions. Recognizing the origin of impulsiveness and procrastination can be key to correct the inefficiencies induced both to the agents themselves and to the individuals with whom they interact. This research is the first attempt to study explicit interpersonal relations of individuals with intrapersonal conflicts of preferences. We have highlighted that imposing sequential completion of projects or competition between agents may mitigate their innate tendency to delay unpleasant tasks and to rush into attractive but unreasonable ones. By contrast, partnerships may exacerbate their willingness both to procrastinate and to rush. However, much work remains to be done if we want to have a good understanding of the interpersonal relations of agents with time-varying preferences.

Appendix

Appendix 1. Proof of Proposition 1

Part (i). Except for integer problems that will be left out of the analysis, it is formally equivalent to analyze the fraction $\lambda(\in [0,1])$ of the n projects or the probability of undertaking each project. Suppose that self-t undertakes each project with probability λ at date t. Given (A1), $\lambda \in \{0,1\}$ cannot be an equilibrium. Anticipating that each self-t ($t \geq 1$) undertakes the project with probability λ, self-0 is indifferent between undertaking it and not if and only if:

$$\begin{aligned}-e+\beta\delta\pi &= \lambda(-\beta\delta e+\beta\delta^2\pi)+(1-\lambda)\lambda(-\beta\delta^2 e+\beta\delta^3\pi)+\cdots\\ &= (-e+\delta\pi)\frac{\lambda\beta\delta}{1-\delta(1-\lambda)}\end{aligned}$$

rearranging terms, we get $\lambda^* = \frac{1-\delta}{\delta(1-\beta)}\left(\frac{\beta\delta\pi}{e}-1\right) \in (0,1)$.

Part (ii). Consists in two steps.

Step 1. Suppose that the agent cannot complete more than one project per period. In addition suppose that $n-1$ projects have been achieved at date τ_n. Self-τ_n anticipates that each future self ($\tau_n+1, \tau_n+2, \ldots$) will undertake the last one with probability λ_1. Then, self-τ_n is indifferent between completing it in the current period and not if:

$$\begin{aligned}-e+\beta\delta\pi &= \lambda_1(-\beta\delta e+\beta\delta^2\pi)+(1-\lambda_1)\lambda_1(-\beta\delta^2 e+\beta\delta^3\pi)+\cdots\\ &= (-e+\delta\pi)\frac{\lambda_1\beta\delta}{1-\delta(1-\lambda_1)}\end{aligned}$$

Therefore $\lambda_1 = \lambda^*$. By the same reasoning, if $n-2$ project have been completed at date τ_{n-1}, self-τ_{n-1} anticipates that all subsequent selves will complete the next to last project with probability λ_2 before completing the last one with probability λ_1. Then, self-τ_{n-1}

is indifferent between undertaking the project in the current period and not if:

$$\begin{aligned}&-e+\beta\delta\pi+\lambda_1(-\beta\delta e+\beta\delta^2\pi)+(1-\lambda_1)\lambda_1(-\beta\delta^2 e+\beta\delta^3\pi)+\cdots\\&\quad=\lambda_2\big[(-\beta\delta e+\beta\delta^2\pi)+\lambda_1(-\beta\delta^2 e+\beta\delta^3\pi)+\cdots\big]\\&\qquad+(1-\lambda_2)\lambda_2\big[(-\beta\delta^2 e+\beta\delta^3\pi)+\lambda_1(-\beta\delta^3 e+\beta\delta^4\pi)+\cdots+\cdots\big]\end{aligned}$$

Which can be rewritten as:

$$2(-e+\beta\delta\pi)=(-e+\delta\pi)\frac{\lambda_2\beta\delta}{1-\delta(1-\lambda_2)}+[-e+\beta\delta\pi]\frac{\lambda_2\delta}{1-\delta(1-\lambda_2)}$$

Recursively, when $n-k$ projects have been already implemented, the $(n-k+1)^{\text{th}}$ one is completed with probability λ_k that is solution of:

$$k(-e+\beta\delta\pi)=(-e+\delta\pi)\frac{\lambda_k\beta\delta}{1-\delta(1-\lambda_k)}+(k-1)[-e+\beta\delta\pi]\frac{\lambda_k\delta}{1-\delta(1-\lambda_k)} \tag{5}$$

Let $g_k(\lambda)=\frac{\lambda}{k(1-\delta)+\lambda\delta}$. It is decreasing in k and increasing in λ. Moreover $g_k(0)=0$ for all k, $g_1(1)=1$ and $\lim_{k\to+\infty} g_k(\lambda)=0$. From (5), if λ_k is an interior solution, it satisfies:

$$\frac{-e+\beta\delta\pi}{\beta\delta[-e+\delta\pi]}=g_k(\lambda_k)$$

Given (A1), an interior solution for λ_k exists if and only if $\frac{-e+\beta\delta\pi}{\beta\delta[-e+\delta\pi]}<g_k(1)$. So, if we denote by $\tilde{n}$ the largest integer such that $\frac{-e+\beta\delta\pi}{\beta\delta[-e+\delta\pi]}<g_{\tilde{n}}(1)$, then $\lambda_k\in(0,1)$ for all $k\leq\tilde{n}$ and $\lambda_k=1$ for all $k>\tilde{n}$. Moreover, $\lambda_k=k\lambda^*$ for all $k\leq\tilde{n}$, so $\tilde{n}$ is the largest integer below $1/\lambda^*$. Overall, when $n\leq\tilde{n}$, the intertemporal welfare from the perspective of self-0 is:

$$\begin{aligned}U(n)&=\lambda_n(-e+\beta\delta\pi)+\lambda_n[(n-1)(-e+\beta\delta\pi)]+(1-\lambda_n)[n(-e+\beta\delta\pi)]\\&=n[-e+\beta\delta\pi]\end{aligned}$$

However, when $n=\tilde{n}+m$ with $m>0$, the agent completes the m first projects with probability 1 at each period and the $\tilde{n}$ subsequent ones with probabilities $\lambda_{\tilde{n}}>\lambda_{\tilde{n}-1}>\cdots>\lambda_1$. Then, the intertemporal welfare from the perspective of self-0 is:

$$U(n^*+m)=-e+\beta\delta\pi+\delta^{m-1}\tilde{n}[-e+\beta\delta\pi]+\beta\delta[-e+\delta\pi]\frac{1-\delta^{m-1}}{1-\delta}$$

Notice that $U(\tilde{n}+m)$ is increasing in m. Besides, $U(\tilde{n}+1)=(\tilde{n}+1)[-e+\beta\delta\pi]$ and $U(\tilde{n}+m)<(\tilde{n}+m)[-e+\beta\delta\pi]$ for all $m>1$.

Step 2. Under no sequentiality restriction and given part (i), the agent undertakes each project at each date with probability λ^*. The intertemporal welfare from the perspective of self-0 in that case is $n[-e+\beta\delta\pi]$. Therefore, when $n\leq\tilde{n}+1=n^*$, there is no loss of welfare from the perspective of self-0 to complete projects sequentially.

Appendix 2. Proof of Proposition 2

Part (i). Suppose that agent 2 undertakes his project at each date with probability p. Besides, self-0 of agent 1 anticipates that each of his future selves $t \geq 1$ will undertake the project with probability q. Self-0 is then indifferent between doing the project in the current period and not if:

$$\begin{aligned} -e + \beta\delta\pi &= (1-p)q(-\beta\delta e - \beta\delta^2\pi) + (1-p)^2(1-q)q \\ &\quad \times (-\beta\delta^2 e + \beta\delta^3\pi) + \cdots \\ &= (-e + \delta\pi)\frac{\beta\delta(1-p)q}{1-\delta(1-p)(1-q)} \end{aligned}$$

which can be rewritten as:

$$q = f(p) \equiv \frac{1-\delta(1-p)}{\delta(1-p)}\Pi \quad \text{where } \Pi = \frac{1}{1-\beta}\left(\frac{\beta\delta\pi}{e} - 1\right)$$

Note that $f(0) = \lambda^* \in (0, 1)$. Besides, $\lim_{p\to 1} f(p) = +\infty$, $f'(p) = \frac{1}{\delta(1-p)^2}\Pi > 0$ and $f''(p) > 0$.

- If $\Pi > \delta$, then $f'(0) > 1$. Given $f''(p) > 0$, we have $f(p) > p$ for all p. In that case, in the unique symmetric equilibrium both agents do the task at the first date.
- If $\Pi < \delta$, then $f'(0) < 1$. Denote $\hat{p}$ the value such that $f'(\hat{p}) = 1$. We have:

$$\frac{1}{\delta(1-\hat{p})^2}\Pi = 1 \Leftrightarrow \hat{p} = 1 - \sqrt{\Pi/\delta}$$

After some manipulations, we get:

$$f(\hat{p}) > \hat{p} \Leftrightarrow (1+\Pi)^2\delta < 4\Pi$$

Given $\Pi < \delta$, this is true if $\Pi > \Pi^* = \frac{2-\delta-2\sqrt{1-\delta}}{\delta}$ (where $\Pi^* \in (0, \delta)$).

Overall, when $e < e^* = \frac{\beta\delta^2\pi}{(1-\beta)(2-\delta-2\sqrt{1-\delta})+\delta}$ (where $e^* \in (\underline{e}, \bar{e})$) so that $\Pi > \Pi^*$, then $\hat{p} < f(\hat{p})$. Hence, $f(p) > p$ for all p and, just as before, both agents do the project in the first period.

Last, when $e \in [e^*, \bar{e})$ so that $\Pi \in (0, \Pi^*)$, then there are two cutoffs (μ, μ') such that $\mu = f(\mu)$ and $\mu' = f(\mu')$ with $\mu < \mu'$. If $p \in [0, \mu) \cup (\mu', 1]$, then $f(p) > p$ and if $p \in (\mu, \mu')$ then $f(p) < p$. The same reasoning holds for the other agent ($p = f(q)$). Hence, in that case, both agents doing the project at each date with probability $p \in \{\mu, 1\}$ are the two stable symmetric equilibria of this game.[14] Note also that $\lambda^* = f(0) < \mu$ since $f'(p) > 0$.

Part (ii). When one agent has already completed his own project, the other is in the same situation as in Proposition 1(i). Therefore, in that case, each self undertakes the project with probability λ^*. Suppose that agent 2 completes his project with probability q (<1) at each period as long as his team mate has not fulfilled his own job. Self-0 of agent 1 anticipates that each of his future incarnations will undertake the activity with probability λ before agent 2 completes his part of the project (and with probability λ^* afterwards). Then, he is indifferent between completing it today and not if and only if:

$$\begin{aligned}
&-e+\beta\delta q\pi+\beta\delta^2\frac{(1-q)\lambda^*\pi}{1-\delta(1-\lambda^*)}\\
&\quad=q(-e+\beta\delta\pi)+\beta\delta\frac{(1-q)q\lambda[-e+\delta\pi]}{1-\delta(1-\lambda)(1-q)}+\beta\delta\frac{-e(1-q)^2\lambda}{1-\delta(1-\lambda)(1-q)}\\
&\qquad+\frac{\lambda^*\beta\delta^3\pi}{1-\delta(1-\lambda^*)}\frac{(1-q)^2\lambda}{1-\delta(1-\lambda)(1-q)}+\frac{\lambda^*[-e+\delta\pi]\beta\delta^2}{1-\delta(1-\lambda^*)}\frac{(1-q)(1-\lambda)q}{1-\delta(1-\lambda)(1-q)} \qquad (6)
\end{aligned}$$

Since we look for a symmetric equilibrium, let $q=\lambda$. Then, after some calculations, the expression reduces to:

$$\begin{aligned}
-e[1-\delta(1-\lambda)^2] = \Bigg[&(1-\lambda)\lambda\delta[-e+\beta\delta\pi]+\lambda\beta\delta[-e+\lambda\delta\pi]\\
&-\frac{\lambda^*\beta\delta}{1-\delta(1-\lambda^*)}\pi\delta[1-\delta(1-\lambda)]\Bigg]
\end{aligned}$$

The potential solution (denoted γ) then satisfies:

$$\frac{\gamma\beta\delta}{1-\delta(1-\gamma)}=\left[\frac{\lambda^*\beta\delta^2\pi}{1-\delta(1-\lambda^*)}-e\right]\frac{1}{-e+\delta\pi}$$

which, using the definition of λ^*, can be rewritten as:

$$\gamma=\frac{1-\delta}{\delta(1-\beta)}\left[\frac{\beta\delta^2\pi^2}{-e(e-2\delta\pi)}-1\right]<\lambda^*$$

Note that:

$$\frac{\beta\delta^2\pi^2}{e(2\delta\pi-e)}>1\Leftrightarrow(1-\beta)<g(e)\equiv\left(1-\frac{e}{\delta\pi}\right)^2$$

Besides, $g'(e)<0$, $g(\bar{e})<(1-\beta)$ and $g(\underline{e})>(1-\beta)$ if and only if $1-2\delta+\beta\delta^2<0$.

Last, note that by (6) if self-0 of agent 1 anticipates $q=0$ for agent 2 and $\lambda=0$ for all his future selves, then he undertakes the project in the current period with probability 1 if $g(e)>(1-\beta)$ and with probability 0 if $g(e)<(1-\beta)$.

Combining all these results, we end up with two cases:

- When $1 - 2\delta + \beta\delta^2 < 0$ (i.e. when $\delta > 1/(1 + \sqrt{1 - \beta})$), there exists a solution $e^{**} \in (\underline{e}, \bar{e})$ such that $(1 - \beta) = g(e^{**})$. For all $e \in [\underline{e}, e^{**})$, $g(e) > (1 - \beta)$ so that being first to undertake the project with probability $\gamma \in (0, 1)$ is the unique stable, symmetric equilibrium. For all $e \in [e^{**}, \bar{e}]$, $g(e) \leq (1 - \beta)$ so that undertaking the project first with probability 0 is the unique stable, symmetric equilibrium.
- When $1 - 2\delta + \beta\delta^2 \geq 0$, then $g(e) \leq (1 - \beta)$ for all $e \in [\underline{e}, \bar{e}]$ and again undertaking the project first with probability 0 is the unique, symmetric equilibrium.

Appendix 3. Proof of Proposition 3

Part (i). From the proof of Lemma 1 and given (A2), we note that an MPE must satisfy $B(c^*; x) = W(c^*)$ where:

$$B(c^*; x) \equiv x[(1 - \delta) + \delta\phi(c^* - m)(1 - \beta)]$$
$$W(c^*) \equiv \beta\delta[c^*(1 - \delta) + \delta\phi(c^* - m)c^* - \delta\phi(c^* - m)m + \delta\varphi(c^* - m)]$$

with $\varphi(\cdot)$ and $\phi(\cdot)$ being the density and c.d.f. of the standard normal distribution. Note that $B_c(c^*; x) > 0, B_{cc}(c^*; x) > 0$ if $c^* < m$ and $B_{cc}(c^*; x) < 0$ if $c^* > m$. Also, $W'(c^*) > 0$ and $W''(c^*) > 0$. Besides, $\lim_{c\to-\infty} B(c) - W(c) > 0$ and $\lim_{c\to+\infty} B(c) - W(c) < 0$. Hence, there can be *at most* three values $c^* \in \{c_1, c_2, c_3\}$ such that $B(c^*; x) = W(c^*)$ where $c_1 < c_2 < c_3$.[15] When this is the case, $[c_1]$ and $[c_3]$ are stable MPEs, while $[c_2]$ is an unstable one. Suppose that, for some parameters, one MPE is such that $c^* = \hat{c} \equiv x/\beta\delta$. This would imply:

$$B(\hat{c}; x) = W(\hat{c}) \Leftrightarrow \frac{\varphi(x/\beta\delta - m)}{\phi(x/\beta\delta - m)} = |m - x/\delta|$$

From the properties of the normal distribution, we know that the Mill ratio $\varphi(y)/\phi(y)$ satisfies: $(\varphi(y)/\phi(y))' < 0$ and $\varphi(y)/\phi(y) \sim_{-\infty} -y$. In our case, this implies: $\lim_{m\to+\infty} \varphi(x/\beta\delta - m)/\phi(x/\beta\delta - m) + x/\delta - m < 0$. Hence, there always exists a value $m^*(>0)$ s.t. $\varphi(x/\beta\delta - m^*)/\phi(x/\beta\delta - m^*) = |m^* - x/\delta|$. This proves that, for a suitably chosen m, $[\hat{c}]$ can be an MPE. Now, given this definition of m^*, three cases are possible depending on (x, β, δ):

(a) $x/\beta\delta - x/\delta = \varphi(0)/\phi(0) \Rightarrow m^* = x/\beta\delta \Rightarrow B''(x/\beta\delta) = 0$. Then, $c_2 = x/\beta\delta$, and therefore $c_1 < \hat{c}$ and $c_3 > \hat{c}$.
(b) $x/\beta\delta - x/\delta < \varphi(0)/\phi(0) \Rightarrow m^* > x/\beta\delta \Rightarrow B''(x/\beta\delta) > 0$. Then, $c_1 = x/\beta\delta$, and therefore $c_2 > \hat{c}$ and $c_3 > \hat{c}$.
(c) $x/\beta\delta - x/\delta > \varphi(0)/\phi(0) \Rightarrow m^* < x/\beta\delta \Rightarrow B''(x/\beta\delta) < 0$. Then, $c_3 = x/\beta\delta$, and therefore $c_1 < \hat{c}$ and $c_2 < \hat{c}$.

Summing up, depending on the parameters, there might be either rush in one of the stable MPEs (case (a)), or rush in both stable MPEs (case (b)) or no rush in any of the stable MPEs (case (c)). Naturally, the same reasoning can be extended to $m \neq m^*$.

When the different selves coordinate on a given MPE $[c^*]$ then, by construction of c^*, the welfare from the perspective of self-0 is:

$$F(c^*)[x - \beta\delta E[c|c \leq c^*]] + (1 - F(c^*))[x - \beta\delta c^*]$$

In the normal case this writes as:

$$x - \beta\delta[c^* + \phi(c^* - m)m - \varphi(c^* - m) - \phi(c^* - m)c^*]$$

which is decreasing in c^* given that $\varphi'(x) = -x\varphi(x)$. Therefore, self-0 is better-off when all his future incarnations coordinate on the smallest stable MPE.

Part (ii). When projects have to be undertaken sequentially and one of them has already been approved, the agent is in the same situation as in (i). Therefore, he undertakes the second one at each period when the cost is below c^*. Anticipating that the MPE for the completion of the last project is $[c^*]$, then $[\tilde{c}]$ is an MPE for the completion of the first project if and only if:

$$x - \beta\delta\tilde{c} + x - \beta\delta c^* = \beta\delta F(\tilde{c})\frac{x - \delta E[c|c \leq \tilde{c}]}{1 - \delta(1 - F(\tilde{c}))} + (x - \beta\delta c^*)\frac{F(\tilde{c})\delta}{1 - \delta(1 - F(\tilde{c}))}$$

which can be rewritten as:

$$B(\tilde{c}; x) + (1 - \delta)[x - \beta\delta c^*] = W(\tilde{c})$$

By definition of $B(\cdot; x)$ and $W(\cdot)$, it comes immediately that $\tilde{c} < c^*$ if $c^* > \hat{c}$ and $\tilde{c} > c^*$ if $c^* < \hat{c}$.

Appendix 4. Proof of Proposition 4

Part (i). Suppose as in the previous section that only the first politician to undertake the project reaps some benefits (and for simplicity if both of them undertake their project simultaneously they both get the full benefit). By analogy with the case of independent projects, if politician a anticipates that politician b will propose his own project whenever the cost is $c \leq c_b^*$, then it is in a's interest to propose it when $c \leq c_a^*$, where:

$$x - \beta\delta c_a^* = \beta\delta F(c_a^*)(1 - F(c_b^*))\frac{x - \delta E[c|c \leq c_a^*]}{1 - \delta(1 - F(c_a^*))(1 - F(c_b^*))}$$

which, rearranging terms, gives:

$$x\left[\frac{1}{1-F(c_b^*)}-\delta+\delta F(c_a^*)(1-\beta)\right]$$
$$=\beta\delta\left[c_a^*\left(\frac{1}{1-F(c_b^*)}-\delta\right)+\delta F(c_a^*)\left(c_a^*-E[c|c\leq c_a^*]\right)\right]$$

Given (A2), this can be rewritten as $\widetilde{B}(c_a^*, c_b^*; x) - \widetilde{W}(c_a^*, c_b^*) = 0$ with:

$$\widetilde{B}(c, c_b^*; x) \equiv x\left[\frac{1}{1-F(c_b^*)}-\delta+\delta\phi(c-m)(1-\beta)\right]$$
$$W(c, c_b^*) \equiv \beta\delta\left[c\left(\frac{1}{1-F(c_b^*)}-\delta\right)+\delta(c-m)\phi(c-m)+\delta\varphi(c-m)\right]$$

Therefore:

$$\frac{\partial c_a^*}{\partial c_b^*} \propto \left.\frac{\partial[\widetilde{B}(c, c_b^*) - \widetilde{W}(c, c_b^*)]}{\partial c_b^*}\right|_{c=c_a^*}$$

So, $\partial c_a^*/\partial c_b^* > 0$ if $c_a^* < \hat{c}$ and $\partial c_a^*/\partial c_b^* < 0$ if $c_a^* > \hat{c}$. Overall, under competition for projects there are as before at most three cutoffs $c^{**} \in \{c_1^{**}, c_2^{**}, c_3^{**}\}$ such that $\widetilde{B}(c^{**}, c^{**}; x) = \widetilde{W}(c^{**}, c^{**})$. Noting that $\widetilde{B}(c, c_b^*; x) = B(c; x)$ and $\widetilde{W}(c, c_b^*) = W(c)$ when $F(c_b^*) = 0$, then one can see that $c^{**} > c^*$ if $c^* < \hat{c}$ and $c^{**} < c^*$ if $c^* > \hat{c}$.

Part (ii). For simplicity, assume that politicians undertake their projects sequentially.[16] The first politician enjoys a private benefit x at the date at which he embarks on his own project, suffers the cost c the period after, and benefits from a positive externality α when the second politician undertakes his project. The politician who invests second enjoys both the private benefit x and the externality α generated by the project already developed in the other jurisdiction the date at which he embarks on his own one, and suffers the cost c the period after investing. Suppose that a moves first. When a's project has been undertaken, b invests if the cost is smaller than $\tilde{c}$ such that:

$$x+\alpha-\beta\delta\tilde{c}=\beta\delta F(\tilde{c})\frac{x+\alpha-\delta E[c|c\leq\tilde{c}]}{1-\delta(1-F(\tilde{c}))}$$

Naturally, $\tilde{c} > c^*$. Self-0 of politician a rationally anticipates that politician b will invest if his cost is smaller than $\tilde{c}$ as soon as he undertakes his project. Let $\check{c}$ be the cost below which politician a invests, then $\check{c}$ is given by:

$$x-\beta\delta\check{c}+\frac{\beta\delta\alpha F(\tilde{c})}{1-\delta(1-F(\tilde{c}))}=\beta\delta F(\check{c})\frac{x-\delta E[c|c\leq\check{c}]}{1-\delta(1-F(\check{c}))}+\frac{\beta\delta F(\check{c})}{1-\delta(1-F(\check{c}))}\frac{\alpha\delta F(\tilde{c})}{1-\delta(1-F(\tilde{c}))} \quad (7)$$

Rearranging terms, we get that $[\check{c}]$ is an MPE if:

$$B(\check{c}; x) + \frac{\beta\delta\alpha(1-\delta)F\check{c}}{1-\delta(1-F(\tilde{c}))} = W(\check{c})$$

As a consequence, $\check{c} > c^*$. Moreover, $B(c; x+\alpha) - [B(c; x) + \frac{\beta\delta\alpha(1-\delta)F(\tilde{c})}{1-\delta(1-F(\tilde{c}))}] > 0$ for all c, then $\check{c} < \tilde{c}$.

From (7), we have $\frac{\partial \tilde{c}}{\partial \alpha} = \frac{B_x(\tilde{c}; x+\alpha)}{W'(\tilde{c}) - B_c(\tilde{c}; x+\alpha)}$. Noting that $B(c; x+\alpha) = \frac{x+\alpha}{\beta\delta} W'(c) - \beta(x+\alpha)\delta\phi(c-m)$ and $B(c; x+\alpha) = (x+\alpha)B_x(c; x+\alpha)$, we obtain $\frac{\partial \tilde{c}}{\partial \alpha} = \frac{1}{\beta\delta + G(\tilde{c})}$ where $G(\tilde{c}) = \frac{x+\alpha}{B(\tilde{c}; x+\alpha)}[\beta^2\delta^2\phi(\tilde{c}-m) - B_c(\tilde{c}; x+\alpha)]$. Consider the function $H(m) = \beta^2\delta - (x+\alpha)(1-\beta)\frac{\varphi(\tilde{c}-m)}{\phi(\tilde{c}-m)}$, then the sign of $G(\tilde{c}(m))$ is the same as that of $H(m)$. Differentiating the equilibrium condition relative to $\tilde{c}$ with respect to m, we get immediately $\frac{\partial \tilde{c}}{\partial m} - 1 < 0$, then $H'(m) < 0$. Moreover, $\lim_{m \to -\infty} H(m) > 0$ and $\lim_{m \to +\infty} H(m) < 0$. Therefore, there exists $\tilde{m}$ such that for all $m > \tilde{m}$, $\frac{\partial \tilde{c}}{\partial \alpha} > \frac{1}{\beta\delta}$. The payoff of politician b decreases when the projects are complements if $x + \alpha - \beta\delta\tilde{c} < x - \beta\delta c^*$. This occurs when $[\tilde{c} - c^*] > \frac{\alpha}{\beta\delta}$. Then, when $m > \tilde{m}$, the overall payoff of b decreases.

Moreover, for all β and δ, there exists $\tilde{x}(\beta, \delta)$ such that $\lim_{\beta \to 0} \tilde{x}(\beta, \delta) = \delta\frac{\varphi(0)}{\phi(0)}$ and for all $x < \tilde{x}(\beta, \delta)$, we have $\frac{x}{\beta\delta} - \frac{x}{\delta} < \frac{\varphi(0)}{\phi(0)}$. Then, there exists $m^*(x, \beta, \delta)$ such that (i) $\frac{\varphi(x/\beta\delta - m^*)}{\phi(x/\beta\delta - m^*)} = m^* - x/\delta$; (ii) $m^*(x, \beta, \delta) > \frac{x}{\beta\delta} = \hat{c}$; (iii) $\hat{c}$ is a MPE; (iv) $\tilde{c} > \hat{c}$ and (v) $\lim_{\beta \to 0} m^*(x, \beta, \delta) = +\infty$. Then, for all m sufficiently large, there exist β, δ and x such that an equilibrium with rush exists.

Similarly, the payoff of politician a decreases when the projects are complements if $\frac{x + \alpha\beta\delta F(\tilde{c})}{1-\delta(1-F(\tilde{c}))} - \beta\delta\check{c} < x - \beta\delta c^*$, i.e. if $\check{c} - c^* > I(\tilde{c})$ where $I(\tilde{c}) = \alpha\frac{F(\tilde{c})}{1-\delta(1-F(\tilde{c}))}$. Moreover, $\frac{\partial \check{c}}{\partial \alpha} = \beta\delta(1-\delta)\frac{I'(\tilde{c})}{W'(\check{c}) - B_c(\check{c}; x)}\frac{\partial \tilde{c}}{\partial \alpha}$. The sign of $W'(\check{c}) - B_c(\check{c}; x)$ is the same as that of $J(m) = \beta\delta - x(1-\beta)\frac{\varphi(\check{c}-m)}{\phi(\check{c}-m)}$. Differentiating the equilibrium condition relative to $\check{c}$ with respect to m, we get $\frac{\partial \check{c}}{\partial m} - 1 < 0$, then $J'(m) < 0$. Moreover, $\lim_{m \to -\infty} J(m) > 0$ and $\lim_{m \to +\infty} J(m) < 0$. Therefore, there exists $\check{m}$ such that for all $m > \check{m}$, $W'(\check{c}) - B_c(\check{c}; x) < \beta\delta(1-\delta)$, in which case $\frac{\partial \check{c}}{\partial \alpha} > I'(\tilde{c})\frac{\partial \tilde{c}}{\partial \alpha}$. Then, when $m > \check{m}$, the payoff of politician a decreases with α.

This proves that if m is sufficiently large and both agents rush under independent projects, then this tendency will be exacerbated under project complementarity.

Notes

1. See Ainslie (1975, 1992), Thaler (1981), Benzion, Rapoport and Yagil (1989), Mazur (1987), Loewenstein and Prelec (1992) and Bleichrodt and Johanesson (2000) for some empirical investigations and theoretical discussions of this phenomenon. See also Mulligan (1996) for a criticism of this approach.
2. Naturally, $\beta = 1$ is the standard case with time consistent preferences. As β decreases, the intra-personal conflict of preferences becomes more important. This quasi-hyperbolic discounting has subsequently been used in most of the previously mentioned papers and also in Laibson (1996, 1997) and Carrillo and Mariotti (2000) among others.
3. There is no loss of generality by assuming that costs and benefits are deterministic, stationary and with a lag of exactly one period.

4. Formally, if in the absence of restrictions the agent is willing to undertake a fraction of projects greater than $1/\lambda^* + 1$, then under sequentiality the utility of the agent will be reduced (see the proof of Proposition 1 for the derivation of this cutoff fraction).
5. An alternative modeling would be to assume that in the case of simultaneous completion, each agent gets $\pi/2$. The results would be essentially the same, but the calculations are more intricate. Also, it would be more natural to assume a stochastic date at which projects are finished. Again, our main results would not be modified. Last, note that the extension to more than two agents is trivial.
6. Formally, there always exists a value $\tilde{e}$ such that for all $e \in (\tilde{e}, \bar{e})$, then $-e + \beta\delta^2\pi < 0$. Note that, in this case, there is not even an asymmetric equilibrium in which the project is completed in equilibrium.
7. Again, the conclusion that severe welfare losses may only require a mild self-control problem was also reached by O'Donoghue and Rabin (1999b) for partially naïve individuals.
8. None of our results change if we rather assume that the uncertainty is on the benefit or both on the cost and the benefit. Setting the variance equal to one is not necessary. However, it simplifies notations considerably.
9. Brocas and Carrillo (1999) consider a finite horizon model. The subgame Perfect equilibrium of their game is therefore unique, it can be computed by backward induction, and depending on the parameters it will correspond to the equilibrium in which the agent does or does not rush.
10. A related result can be found in Carrillo (1998). Note that patience is valuable independently of whether there is rush or not: when two MPEs coexist it is ex ante optimal for every self to coordinate on the one that specifies the lowest cutoff c^*.
11. Recall from Proposition 3(i) that the MPE specifying the lowest cutoff provides always the greatest utility to the agent.
12. For a comprehensive theoretical and empirical discussion of this application see Della Vigna and Paserman (2000).
13. A related point is made in Caillaud, Cohen and Jullien (1996).
14. Both agents completing the task with probability μ' is also a symmetric equilibrium, although unstable.
15. Note also that, if $\beta = 1$, we have $B'(c^*; x) = 0$. Then, for any distribution, there is one and only one MPE $[c^*]$. Furthermore, $x - \delta c > 0$ for all $c < c^*$ so that, as it is well known, under time consistency rush never occurs.
16. The analysis could be extended to situations in which both politicians can embark on their projects at the same date but computations become much more intricate.

References

Ainslie, George. (1975). "Specious Reward: a Behavioral Theory of Impulsiveness and Impulse Control," *Psychological Bulletin* 82, 463–496.

Ainslie, George. (1992). *Picoeconomics*. Cambridge University Press.

Akerlof, George. (1991). "Procrastination and Obedience," *American Economic Review* 81, 1–19.

Benzion, Uri, Amnon Rapoport, and Joseph Yagil. (1989). "Discount Rates Inferred from Decisions: An Experimental Study," *Management Science* 35, 270–284.

Bleichrodt, Han, and Magnus Johanesson. (2000). "Time Preference for Health: a Test of Stationarity versus Decreasing Timing Aversion," 44, 1104–1115.forthcoming in the *Journal of Mathematical Psychology*.

Brocas, Isabelle, and Juan D. Carrillo. (1999). "A Theory of Haste with Applications to Impulse Buying and Destruction of the Environment," CEPR D.P. 2027.

Brocas, Isabelle, and Juan D. Carrillo. (2000). "The Value of Information when Preferences are Dynamically Inconsistent," *European Economic Review* 44, 1104–1115.

Caillaud, Bernard, Daniel Cohen, and Bruno Jullien. (1996). "Towards a Theory of Self-Restraint," *mimeo*, Paris.

Carrillo, Juan D. (1998). "Self-Control, Moderate Consumption, and Craving," CEPR D.P. 2017.

Carrillo, Juan D., and Thomas Mariotti. (2000). "Strategic Ignorance as a Self-Disciplining Device," *Review of Economic Studies* 67, 529–544.

DellaVigna, Stefano, and Daniele Paserman. (2000). "Job Search and Hyperbolic Discounting," *mimeo*, Harvard University.

Laibson, David. (1996). "Hyperbolic Discount Functions, Undersaving, and Savings Policy," *mimeo*, Harvard University.

Laibson, David. (1997). "Golden Eggs and Hyperbolics Discounting," *Quarterly Journal of Economics* 112, 443–477.

Loewenstein, George, and Drazen Prelec. (1992). "Anomalies in Intertemporal Choice: Evidence and an Interpretation," *Quarterly Journal of Economics* 107, 573–598.

Mazur, James E. (1987). "An Adjustment Procedure for Studying Delayed Reinforcement." In M. Commons, J. Mazur, J. Nevins, and H. Rachlin (eds.), *Quantitative Analysis of Behavior: The Effect of Delay and of Intervening Events on Reinforcement Value*. Hillsdale: Ballinger.

Mulligan, Casey. (1996). "A Logical Economist's Argument Against Hyperbolic Discounting," *mimeo*, University of Chicago.

O'Donoghue, Ted, and Matthew Rabin. (1999a). "Doing it Now or Later," *American Economic Review* 89, 103–124.

O'Donoghue, Ted, and Matthew Rabin. (1999b). "Choice and Procrastination," *mimeo*, UC Berkeley.

Phelps, Edmund, and Robert A. Pollak. (1968). "On Second Best National Saving and Game-Theoretic Growth," *Review of Economic Studies* 35, 185–199.

Rook, Dennis W. (1987). "The Buying Impulse," *Journal of Consumer Research* 14, 189–199.

Strotz, Robert H. (1956). "Myopia and Inconsistency in Dynamic Utility Maximisation," *Review of Economic Studies* 23, 166–180.

Thaler, Richard. (1981). "Some Empirical Evidence on Dynamic Inconsistency," *Economics Letters*, 201–207.