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ENDOGENOUS ENTRY IN AUCTIONS WITH NEGATIVE EXTERNALITIES

ABSTRACT. In this paper, we study the auction to allocate an indivisible good when each potential buyer has a private and independent valuation for the item and suffers a negative externality if a competitor acquires it. In that case, the outside option of each buyer is mechanism-dependent, which implies that participation is endogenous. As several works in the literature have shown, the optimal auction entails strong threats to induce full entry and maximal expected revenue. This results from the full commitment assumption, which ensures that threats are credible. We show that absent credible threats, the entry process does not lead to full participation: the equilibrium entails screening of agents in the entry stage and a trade-off between reserve prices and entry fees. Besides, we discuss the conditions under which the impossibility to use threats does not prevent the seller from ensuring a minimal screening and reaching a high expected revenue.

KEY WORDS: Auctions, Commitment, Coordination, Externalities

1. INTRODUCTION

When agents compete for the acquisition of an item, they may suffer negative externalities induced by other (winning) competitors. This is the case when a public project is delegated to a firm or when an employee is selected to undertake a mission. Recently, privatizations in Eastern Europe also exhibited the presence of negative externalities suffered by past workers who attempted to buy the firms to keep their jobs. Also, when several firms want to become licensees of a new technology, each firm wants not only to increase his own probability of being the winner but, at the same time, to decrease the probability that a competitor gets the license. Naturally, the seller who organizes the tournament can benefit from tight competition. The presence of negative externalities modifies price making and the strategic behavior of all the parties involved. One of the most relevant framework able to provide a characterization and an explanation of the changes induced by the presence of externalities in a compet-

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itive setting is auction theory.[1] Auction theory has been developed to provide an explicit model of price making in a context of uncertainty and strategic behavior under asymmetric information.[2] In this paper, we provide a model of auctions with negative externalities and derive a comparison with the familiar results of the literature.

Negative externalities are exerted (by the winner on losers) only when the good is sold. For each potential buyer, there are three states of nature: (a) no one gets the good, (b) he gets it, and (c) another one gets it. This has two main consequences. First, the expected outcome of a looser and a non participant is uncertain and, participation constraints become non trivial. Indeed, the outside option of any potential bidder is *mechanism-dependent* since it depends on the allocation rule that is used when he does not participate. Second, participants are ready to 'overbid', that is pay more than the intrinsic value of the item.

A number of analyses of negative externalities have been proposed. Several have been developed in the context of patent licensing. The papers by Katz and Shapiro (1986) and Kamien and Tauman (1986) show that in the presence of interdependent demands (which reflects the presence of externalities) the optimal strategy of the seller when potential buyers are identical is not the standard price mechanism. Kamien et al. (1992) highlight the fact that the patent-holder can extract some surplus from non-acquirers.[3] Closer to our work is the recent development of the theory of auctions with externalities. Jehiel et al. (1996)[4] suggest a model of auction with observable and (particular) identity-dependent externalities[5] in which the seller can commit. They show that the optimal auction is such that an agent (possibly a third party) gets the good with probability one when at least one potential bidder does not participate. If all show up, the allocation rule involves a reserve price and the good is not necessarily sold. Then, each potential bidder is ready to pay to participate to buy himself a chance of not suffering the externality. As a result, the optimal auction is implemented by (i) anonymous auctions with reserve prices and entry fees when all bidders participate and (ii) a threat to induce full participation.

Since threats imply the resort to a third party or the commitment to give the good for free to one buyer when nobody participates, it may happen that (i) the seller cannot commit to give the good for free to a third party for institutional reasons, (ii) no third party exists, or (iii) the seller and the buyers prefer to renegotiate the allocation rule if nobody showed up (since they are better-off if nobody gets the good). This questions the issue of the credibility of the procedures that implement the optimal auction. Unless auction theory has been increasingly applied[6] to auction items that incorporate an externality component, the issue of practical implementation has been neglected in the literature. The purpose of this paper is to analyze how commitment abilities affect rent extraction (i.e., revenue maximizing) in competitive settings with negative externalities.

In this paper we consider a dynamic structure for the auction, consisting of an entry stage and a bidding process. From a theoretical perspective, this approach helps us to capture the effects of the mechanism dependency phenomenon on the entry decision. From a practical perspective, agents are selected through a two-step procedure in many economic situations (e.g. calls for projects, job openings, sale of spectrum, licensing). They first decide whether they participate and then compete. We first restate the characterization of the optimal auction under full commitment (Lemma 1 and 2). Then, we relax full commitment, and instead we assume that the seller can only commit not to renegotiate the rules of the auction she offers which consists of an entry fee and a series of reserve prices; that is, she can credibly commit not to organize a subsequent auction if she does not sell the good (which is the standard commitment assumption required to implement optimal auctions without externalities), but she cannot commit to sell the good for sure when some bidders do not participate. Then, the mechanism-dependency phenomenon induces an *endogenous* entry decision.[7] We characterize the bidding strategies of participants in first-price sealed bid auctions (Proposition 1). We show that the equilibrium procedure entails a trade-off between reserve prices and entry fees, both called for by the mechanism dependency of the outside option (Proposition 2). We also show that the solution entails screening of agents in the entry stage, which allows us to stress the importance of the commitment assumption used in the optimal auction literature (Proposition 3). Moreover, potential bidders might face a coordination problem in the entry stage and we discuss the conditions under which the equilibrium is unique (Proposition 4).

The paper is organized as follows. Section 2 analyzes the optimal auction and restates the results obtained in the literature when the seller can commit. Section 3 solves for the case with limited commitment. Section 4 concludes.

2. THE OPTIMAL AUCTION

2.1. *Preliminaries*

An indivisible good is offered for sale among $N \geq 2$ risk neutral potential buyers. The seller (she) offers a mechanism $\mathcal{A}$ to potential buyers. We suppose, without loss of generality, that the seller's valuation for the good is zero. Buyer i derives a utility v_i when he gets the good, which is his valuation for the good or willingness to pay for it. Each v_i is drawn independently from a common knowledge distribution $F(v_i)$, with density $f(v_i)$. $F(\cdot)$ is assumed to be strictly increasing and continuously differentiable on the interval $V = [\underline{v}, \overline{v}]$ with $F(\underline{v}) = 0$ and $F(\overline{v}) = 1$. Moreover, we suppose that the distribution satisfies the following assumption to avoid bunching phenomena:

ASSUMPTION 1.

$$v - \frac{1 - F(v)}{f(v)}$$

is increasing with respect to v.

Bidder i suffers an externality α (> 0) when bidder $j \neq i$ gets the good. We assume that α is common knowledge. [8] If nobody gets the good, the utility of each agent is 0.

2.2. *Auction under commitment*

We begin by restating the result obtained when the seller can commit, and therefore choose any mechanism. We need to note two things. First, given any mechanism $\mathcal{A}$, if agent i does not participate, he will suffer an externality *if an entrant acquires the good*. Hence, his outside option depends on the probability that someone wins when he refuses to participate, i.e., on the auction rule. In other words, the presence of externalities induces a non-trivial participation constraint which is *mechanism-dependent*. Second and obviously, the worst outside option of an agent corresponds to the case in which a competitor gets the good for sure. Combining these two elements, if $\mathcal{A}$ (i) specifies a probability one of suffering the externality in case of non participation and (ii) ensures a utility level at least equal to the outside option in case of participation, then any agent is better off if he shows up. Such mechanism is optimal. Precisely,

LEMMA 1. *(Jehiel et al. (1996)–Brocas (2002b)). The optimal mechanism $\mathcal{A}^*$ entails:*

- *(i) If at least one agent does not participate, the good is allocated for sure (e.g., to one agent, a third party).*
- *(ii) If all agent participate, the agent with the highest valuation gets the good provided his valuation is above the cutoff $r^*(\alpha, N)$ with*

$$r^*(\alpha, N) = \arg\max \left\{ v \middle| v - \frac{1 - F(v)}{f(v)} \geqslant \alpha(N-1) \right\}.$$

Proof. See Jehiel et al. (1996) for the case of identity-dependent externalities (the proof follows by considering the particular case where the externality is α whatever the identity of the competitor). See also Brocas (2002b) for the case of type-dependent externalities of the form $\alpha(v_i, v_j) = \alpha + \alpha_1 v_i + \alpha_2 v_j$ (the proof follows by considering the particular case $\alpha_1 = \alpha_2 = 0$). □

As already noted in the literature, the presence of negative externalities modifies the standard pricing mechanism (which can be derived by posing $\alpha = 0$). First (point (i)), it is optimal for the seller to force full entry and to push agents towards their worst outside option: potential bidders are ready to make a payment up to their outside option to prevent the sale of the item. By pushing agents towards their worst outside option, the seller increase those payments. Second (point (ii)), the optimal reserve price depends on the number of bidders and on the level of the externality: conditional on everybody participating, each agent is ready to pay α to prevent any competitor from getting the good. In other words, the revenue from the sale to agent i is equal to the willingness to pay of that agent net of informational costs, and the revenue of not selling it is equal to the sum of the payments his competitors are ready to make $\alpha(N-1)$. Overall, the *virtual surplus* of the seller is $v_i - (1 - F(v_i))/f(v_i) - \alpha(N-1)$, and the optimal reserve price is the smallest value such that the virtual surplus is positive.

LEMMA 2. *(a) The optimal auction can be implemented by a mechanism such that (i) the good is allocated for sure when at least one bidder does not participate, (ii) participants pay an entry fee* $c^* = \alpha F(r^*(\alpha, N))^{N-1}$ *and (iii) bid in a second price sealed bid auction with reserve price* $r^*(\alpha, N)$*. The optimal bidding strategy is* $b(v_i) = v_i + \alpha$*. (b) The revenue equivalence theorem holds.*

Proof. See Brocas (2002b) for (a).[9] Proving that the revenue equivalence holds (b) follows the same steps as in Myerson (1981) and the proof is omitted: it is a consequence of the revelation principle (which is invoked to characterize the optimal auction). □

The optimal auction can be implemented by a simple mechanism where the seller forces full entry by threatening agents with their worst outside option (to satisfy point (i) in Lemma 1) and designs an anonymous auction. Naturally, given the presence of a reserve price in the optimal auction, the seller must not accept bids below $r^*(\alpha, N)$. Moreover, since agents are ready to pay to prevent their competitors from acquiring the good, it is optimal to set an entry fee. Formally, the utility of an agent with valuation $v_i < r^*(\alpha, N)$ is $-\alpha$ if he does not show up (point (a)(i) in Lemma 2). If he participates to the second price sealed bid auction with reserve price, his expected utility is $-\alpha[1-F(r^*(\alpha, N))^{N-1}]$ and he prefers to enter if the entry fee is less than $\alpha F(r^*(\alpha, N))^{N-1}$. Last, the bidding strategy reflects both the willingness to pay to get the good v_i and the willingness to pay to avoid the sale to a competitor α.

3. AUCTION WITH LIMITED COMMITMENT

Section 2 established that the optimal mechanism requires a strong commitment assumption (point (i) in Lemma 1), basically to be able to sell the good for sure *even if nobody participates*. Full commitment allows the seller to select the worst possible outside option of each agent. In this section, we determine how the outcome is modified when the seller cannot commit to sell the good for sure in case of partial entry. In particular, she cannot resort to a third party if nobody enters. If so, the outside option of each bidder depends on the allocation rule faced by his competitors if he does not participate. [10] Given the seller cannot push agents towards their worst outside option, some agents might decide not to enter. Then, the seller can infer some information about the valuations of entrants. Overall, the problem boils down to a dynamic game of asymmetric information and the revelation principle does not apply. We restrict the attention to a particular class of mechanisms that is presented in the next subsection. As will become clear, our restriction is not ad-hoc since (i) our solution tends to the result in Section 2 if we restore the possibility to threaten, and (ii) our solution also tends to the result obtained when $\alpha = 0$ (see Myerson (1981)) in which case the possibility to threaten is not needed.

3.1. *Preliminaries*

Stage 0 (Pricing mechanism): the seller proposes a mechanism $\mathcal{A}$. It consists of an entry fee c and an allocation rule. We assume that the allocation of the good is made via a first price sealed bid auction [11] where bidders face a reserve price contingent on the number of participants. Reserve prices are summarized by the vector $(r_1, ..., r_N)$.
Stage 1 (Participation decision): each potential buyer decides whether to participate or not. Formally, buyer i chooses a strategy $s(v_i) \in \{e, ne\}$ where $s(v_i) = e$ if i enters into the auction and $s(v_i) = ne$ if i does not enter. If $s(v_i) = e$, then i pays c. Therefore, during the entry process, a subset of potential bidders become actual bidders. Let $I = \{i, i \in \{1, ..., N\}, s(v_i) = e\}$. The number of actual bidders is thus a random variable $Y = \text{card} \quad I$ on $\{1, ..., N\}$. Note $P(\cdot)$ the distribution of Y. At the end of the first period, the seller and each potential bidder learn the realization of Y.
Stage 2 (Auction): the auction takes place with n bidders and the reserve price is r_n. Bidders submit bids and the bidder who makes the highest bid is awarded the good in exchange of a payment (his bid). Besides, we assume there is a common bidding strategy such that the bid of agent i is $b_i = b(v_i)$ where $b(\cdot)$ is increasing in v_i.

Since the submission of a bid requires the expenditure of c, an entrant will not bid at all for valuations below some break-even level $\hat{v}$. An agent with this valuation is indifferent between entering and not and, if he enters, he will win only if no other agent participate. In that case, his best strategy will be to bid the reserve price. We look for a symmetric equilibrium strategy $s(\cdot)$ that is increasing in v_i in the following sense: if for some valuation the agent feels in his interest to enter (resp. not enter), then for any valuation above (resp. below) this value, the agent also decides to enter (resp. not to enter). Denote by:

- $\hat{v}$, the cutoff valuation for entry (or entry rule);
- $u_i^e(v_i, n)$, the expected utility of agent i, with valuation v_i, if he participates when $n - 1$ other agents do;
- $u_i^{ne}(v_i, n - 1)$, the expected utility of agent i if he does not participate, while $n - 1$ other agents do;
- $\pi_n(\hat{v})$, the probability for a given agent that n other bidders have decided to enter;
- $p_n(\hat{v}) = P(Y = n)$, the probability that n agents participate;
- $U_i^e(v_i, \hat{v}; \mathcal{A}) = \Sigma_{n=0}^{N-1} \pi_n(\hat{v}) u_i^e(v_i, n + 1)$, the ex ante expected utility of agent i if he participates when the other agents play the entry rule $\hat{v}$ when the auction mechanism is $\mathcal{A}$;
- $U_i^{ne}(v_i, \hat{v}; \mathcal{A}) = \Sigma_{n=0}^{N-1} \pi_n(\hat{v}) u_i^{ne}(v_i, n)$, the ex ante expected utility of agent i if he does not participate when the other agents play the entry rule $\hat{v}$ and if the auction mechanism is $\mathcal{A}$;
- $H(v_i, \hat{v}; \mathcal{A}) = U_i^e(v_i, \hat{v}; \mathcal{A}) - U_i^{ne}(v_i, \hat{v}; \mathcal{A})$, the differential in utility between participating and not.

Let us introduce some definitions to facilitate the exposition.

DEFINITION 1. (Existence) Given $\mathcal{A}$, $\hat{v} \in (\underline{v}, \overline{v})$ is a rational expectation equilibrium (REE) if and only if:

$$\begin{array}{ll} H(v_i, \hat{v}; \mathcal{A}) - c < 0 & \forall\ v_i < \hat{v} \\ H(v_i, \hat{v}; \mathcal{A}) - c > 0 & \forall\ v_i > \hat{v} \\ H(\hat{v}, \hat{v}; \mathcal{A}) - c = 0 & \end{array}$$

Moreover $\underline{v}$ (resp. $\overline{v}$) is a REE if $H(\underline{v}, \underline{v}; \mathcal{A}) - c \geq 0$ (resp. $H(\overline{v}, \overline{v}; \mathcal{A}) - c \leq 0$).

Hence $\hat{v}$ is such that, $\forall\, v_i \geq \hat{v}$, $s(v_i) = e$ and $\forall\, v_i < \hat{v}$, $s(v_i) = ne$.[12]

DEFINITION 2. (Uniqueness) $\hat{v}$ is a unique REE if and only if there does not exist an other $\hat{\hat{v}}$ that satisfies also Definition 1.

Let $\hat{v}$ and $\hat{\hat{v}}$ be two REE when the seller proposes $\mathcal{A}$. Moreover, denote by $s(v_i, \hat{v})$ the strategy played by agent i when he anticipates that the other agents play the rule $\hat{v}$ and by $U(v_i, \hat{v}; \mathcal{A}|s(v_i, \hat{v}))$ the utility of agent i when he plays according to $s(v_i, \hat{v})$.

DEFINITION 3. (Pareto dominance) $\hat{v}$ is Pareto dominant for the pool of participants if for all $\hat{\hat{v}}$ and for all v_i:

$$U(v_i, \hat{v}; \mathcal{A}|s(v_i, \hat{v})) \geq U(v_i, \hat{\hat{v}}; \mathcal{A}|s(v_i, \hat{\hat{v}})).$$

In the auction stage, agents who have a valuation below $\hat{v}$ do not participate. Given an entry rule $\hat{v}$, the revised distribution of types that is relevant in the auction stage is $F(v|\hat{v})$ where $(F(v) - F(\hat{v}))/(1 - F(\hat{v}))$ for all $v \geqslant \hat{v}$ and $F(v|\hat{v}) = 0$ otherwise.

3.2. *Bidding strategies in the auction stage*

Our first result characterizes the optimal bidding strategy in the auction stage.

PROPOSITION 1. *The optimal bidding (pure) strategy in a first price sealed bid auction with $n > 1$ bidders and reserve price r_n is*

$$b(v_i) = v_i + \alpha - \frac{\int_y^{v_i} F(s|\hat{v})^{n-1} \mathrm{d}s}{F(v_i|\hat{v})^{n-1}} - \frac{\alpha F(y|\hat{v})^{n-1}}{F(v_i|\hat{v})^{n-1}} \quad \text{if} \quad v_i \geq y$$

where $y = \max(r_n, \hat{v})$. It is increasing in the number of bidders.

Proof. See Appendix. □

In an auction with negative externalities, an agent with valuation v_i is ready to pay at most v_i to get the good but also α to prevent his competitors from acquiring it. In a first price auction, the agent has incentives to decrease his bid below $v_i + \alpha$ to take advantage of his private information.[13,14] Last, the higher the number of competitors, the smaller the incentives to decrease the bid below $v_i + \alpha$ and take the risk of losing the good to the benefit of competitors. [15]

3.3. *Optimal allocation*

Suppose that only bidders with valuation above $\hat{v}$ participate and assume that $n > 1$ have entered and paid the entry fee c. Given the bidding strategy in Proposition 1, it is easy to compute the seller's expected revenue $R_n(\hat{v}, r_n)$:

$$\begin{cases} \int_{r_n}^{\overline{v}} n \frac{[F(v_i) - F(\hat{v})]^{n-1}}{[1 - F(\hat{v})]^n} \left[v_i + \alpha - \frac{1 - F(v_i)}{f(v_i)} \right] f(v_i) \mathrm{d}v_i - \\ \quad n\,\alpha \frac{[F(r_n) - F(\hat{v})]^{n-1}}{[1 - F(\hat{v})]^n} [1 - F(r_n)] \quad \text{if } r_n \geq \hat{v} \\ \int_{\hat{v}}^{\overline{v}} n \frac{[F(v_i) - F(\hat{v})]^{n-1}}{[1 - F(\hat{v})]^n} \left[v_i + \alpha - \frac{1 - F(v_i)}{f(v_i)} \right] f(v_i) \mathrm{d}v_i \\ \quad \text{if } r_n < \hat{v} \end{cases}$$

If only one agent has entered, he will not bid above the reserve price r_1 and the expected revenue is $R_1(\hat{v}, r_1) = r_1(1 - F(r_1|\hat{v}))$ if $r_1 \geq \hat{v}$ and $R_1(\hat{v}, r_1) = r_1$ if $r_1 < \hat{v}$.

Conditional on the entry rule $\hat{v}$, the seller determines the optimal procedure $\mathcal{A}$ that maximizes her ex ante (before entry) expected revenue provided $\hat{v}$ is a REE, that is

$$\begin{aligned} &\textstyle\sum_{n=1}^{N} p_n(\hat{v}) \Big(R_n(\hat{v}, r_n) + nc \Big) \\ &\text{s.t.} \quad c = H(\hat{v}, \hat{v}, \mathcal{A}) \end{aligned}$$

The next result characterizes the optimal procedure conditional on the entry rule $\hat{v}$.

PROPOSITION 2. *Suppose the entry rule is $\hat{v} > \underline{v}$. The optimal procedure $\hat{\mathcal{A}}$ entails:*

(i) *if* $n > 1$ *agents participate, the optimal reserve price is* $\hat{r}_n(\hat{v}) = \max(\hat{v}, \tilde{r}_n(\hat{v}))$ *where* $\tilde{r}_n(\hat{v})$ *is solution of*

$$v - \alpha(n-1) - \frac{1-F(v)}{f(v)} = -\alpha(N-n)\frac{1-F(\hat{v})}{F(\hat{v})}.$$

Moreover, if there exists n_1 *such that* $\hat{r}_{n_1}(\hat{v}) = \tilde{r}_{n_1}(\hat{v})$*, then* $\hat{r}_n(\hat{v}) = \tilde{r}_n(\hat{v})\ \forall n > n_1$*. Last,* $\hat{r}_{n+1}(\hat{v}) > \hat{r}_n(\hat{v})$*;*

(ii) *if* $n = 1$*, the optimal reserve price is whatever value* $\hat{r}_1 \in [\underline{v}, \hat{v}]$*;*

(iii) *the entry fee is*

$$\hat{c} = \pi_0(\hat{v})(\hat{v} - \hat{r}_1) + \Sigma_{n=1}^{N-1}\alpha\pi_n(\hat{v})F(\hat{r}_{n+1}(\hat{v})|\hat{v})^n - \Sigma_{n=2}^{N-1}\alpha\pi_n(\hat{v})F(\hat{r}_n(\hat{v})|\hat{v})^n.$$

Last, if the entry rule is $\hat{v} = \underline{v}$*, then the optimal procedure is* $\mathcal{A}^{**}$ *such that* $\hat{r}_N(\underline{v}) \equiv r^*(\alpha, N)$*,* $\hat{r}_n(\underline{v}) = \underline{v}$ *for all* $n < N$ *and* $\hat{c} = \alpha F(\hat{r}_N(\underline{v})|\underline{v})^{N-1} \equiv c^*$.

Proof. See Appendix. □

Several comments are in order. First, the mechanism dependency phenomenon affects the reserve prices compared to the solution obtained under full commitment (Lemma 1). Suppose that agents coordinate on $\hat{v}$. Agents with valuations less than $\hat{v}$ do not enter and get

$$U_i^{ne}(v_i, \hat{v}; \hat{\mathcal{A}}) = -\alpha\sum_{n=2}^{N-1}\pi_n(\hat{v})[1 - F(\hat{r}_n(\hat{v})|\hat{v})^n] - \alpha\pi_1(\hat{v})$$

which is the outside option generated by the procedure. The mechanism dependency implies that if the seller wants to induce a given outside option, she has to adjust the outcomes of the agents in each state of nature. Intuitively, the reserve price she fixes for n participants influences both the outcome of an agent who participates in an auction with $n-1$ others, and his outcome when he does not participate while n others do. Hence, distortions are called for by mechanism-dependency and are interdependent. However, the reserve price for N participants influences only the outcome of the auction in which all potential bidders participate and therefore it is not distorted, and equal to the reserve price obtained in Lemma 1.

Second, reserve prices are as high as the number of actual bidders is high. Indeed, the higher the number of participants, the higher the bids (see Proposition 1). Therefore, if the seller is indifferent between selling the good or not to a bidder who bids $\hat{r}_{n-1}(\hat{v})$ in an auction with $n-1$ participants, she strictly prefers not to sell it at this price in an auction with n bidders.

Third, the reserve price when $n = 1$ is necessarily lower than the entry rule $\hat{v}$. The reason is simply that if $\hat{v} < \hat{r}_1$, all agents with valuations $< \hat{v}$ have the same expected utility than an agent with valuation $\hat{v}$. In other words, $\hat{v}$ is not equilibrium in the entry stage. Moreover, when $n = 1$, the bidder always offers the reserve price and the revenue of the seller is a sure outcome. Given the entry fee is adjusted so that an agent with valuation $\hat{v}$ is indifferent between entering or not (definition 1), the seller is indifferent between capturing more money in the auction with one bidder (by increasing $\hat{r}_1$ and decreasing $\hat{c}$) or in the entry stage (by increasing $\hat{c}$ and decreasing $\hat{r}_1$). Overall $\hat{r}_1$ can take any value in between $\underline{v}$ and $\hat{v}$ and $\hat{c}$ is adjusted accordingly.

Last, suppose agents coordinate on $\hat{v} = \underline{v}$. In that case, $\pi_n(\hat{v}) = 0$ for all $n < N-1$ and $\pi_{N-1}(\hat{v}) = 1$, i.e., there is full entry. Then, the optimal allocation consists in offering the procedure $\mathcal{A}^{**}$ which corresponds simply to $\mathcal{A}^{*}$ less the threat (i.e., Lemma 1, part (ii)).

3.4. *Screening*

Proposition 2 characterizes the optimal allocation rule $\hat{\mathcal{A}}$ corresponding to an arbitrary entry rule $\hat{v}$. By construction, if the seller offers $\hat{\mathcal{A}}$, the entry rule $\hat{v}$ is an equilibrium. The purpose of this section is to check whether the entry rule $\hat{v}$ corresponding to $\hat{\mathcal{A}}$ is unique, that is whether agents unambiguously coordinate on $\hat{v}$ when the seller offers $\hat{\mathcal{A}}$ anticipating they would coordinate on $\hat{v}$.

3.4.1. *Full entry and limited commitment*

If agents coordinate on $\hat{v} = \underline{v}$, Proposition 2 tells us that the optimal allocation consists in offering the procedure $\mathcal{A}^{**}$ such that $\hat{r}_N(\underline{v}) \equiv r^*(\alpha, N)$, $\hat{r}_n(\underline{v}) = \underline{v}$ for all $n < N$ and $\hat{c} = \alpha F(\hat{r}_N(\underline{v})|\underline{v})^{N-1} \equiv c^*$. If agents decide to coordinate on $\hat{v} = \underline{v}$, $\mathcal{A}^{**}$ allows to reach the highest possible payoff, and there is no cost resulting from the seller's lack of commitment power. However, PROPOSITION 3. *For all $\alpha > 0$, when the seller offers $\mathcal{A}^{**}$, the entry rule $\underline{v}$ is a REE but it is not unique and it is never Pareto dominant. In particular, it is dominated by $\overline{v}$ if α is sufficiently large.*

Proof. See Appendix. □

Naturally, $\underline{v}$ is a REE by construction. Indeed, $\mathcal{A}^{**}$ is the optimal procedure associated to the entry rule $\hat{v} = \underline{v}$ and the entry fee is set so that all agents with valuations higher than $\underline{v}$ prefer to show up *conditional on anticipating that all other agents do so*.

However, the pool of agents never prefer to coordinate on $\hat{v} = \underline{v}$. Indeed, if they do so, all agents with valuations less than $r^*(\alpha, N)$ have an expected utility equal to $-\alpha$. However, if they choose to coordinate on $\hat{v} > \underline{v}$, bidders with valuations below this cutoff get a higher utility in equilibrium. In other words, agents with low valuations prefer to coordinate on $\hat{v} > \underline{v}$ rather than on $\hat{v} = \underline{v}$.

Last, the size of the externality matters. It is easy to see in Lemma 1 that the optimal reserve price is $\overline{v}$ when α is large. In that case, the seller commits to sell the good for sure if at least one agent does not show up, specifies a large entry fee and never sells the good conditional on full entry. Naturally, given the threat, agents participate, they all pay the fee and nobody gets the good. If the threat is absent, bidders anticipate that the good will not be sold if none of them enter and prefer to save the large entry fee. In equilibrium, they decide not to show up, none of them acquire the good and no one suffers the externality.

To sum up, we have shown that $\mathcal{A}^{**}$ is optimal provided that all agents enter (see Proposition 2). However, given the absence of commitment, agents can decide to coordinate on a different entry rule (see Proposition 3). Overall, the solution is likely to entail strategic non-participation, which results in a screening of agents: low valuation bidders do not show up and high valuation agents participate.

It is important to note that the previous result helps us to understand the role of full commitment in optimal auctions. If commitment is limited in the sense that threats are not available, agents are likely to deviate from full entry, which results in an unexpectedly lower revenue. In the limit case where externalities are large and agents eventually coordinate on $\hat{v} = \overline{v}$ rather than $\hat{v} = \underline{v}$, the revenue of the seller is 0.

3.4.2. *Strategic non participation and uniqueness*

Going back to the general case, our last result is as follows:

PROPOSITION 4. *When α is sufficiently small, the seller can select a procedure $\hat{\mathcal{A}}$ corresponding to an entry rule $\hat{v}$ close to $\underline{v}$ such that $\hat{v}$ is the unique equilibrium of the entry game. By contrast, when α is sufficiently large, $\overline{v}$ is always an equilibrium of the entry game whatever the procedure that is offered.*

Proof. See Appendix. □

The level of the externality affects the equilibrium outcome. In the limit case $\alpha = 0$, if the seller offers the reserve prices defined in Proposition 2 (i.e., $\mathcal{A}^{**}$ for $\alpha = 0$), the unique equilibrium in the entry stage is $\underline{v}$. Naturally, this is the case because the auction boils down to a standard auction without externality, which does not require threats to be optimally implemented. When $\alpha > 0$ (even small), $\underline{v}$ is not the unique equilibrium anymore when the seller offers $\mathcal{A}^{**}$ (see Proposition 3). However, as long as α is small, it is possible to induce the participation of most of the bidders. As α increases, the option of not participating becomes more valuable. Each agent anticipates that his competitors might decide not to show up, in which case the good is not sold and no externality is suffered. Then, it becomes costly for the seller to induce participation (in particular the entry fee needs to be reduced). When α is sufficiently large (e.g. larger than the benefit of the good itself), agents have always the incentives to coordinate on $\overline{v}$. In that case, if the seller offers the procedure $\hat{\mathcal{A}}$ corresponding to the entry rule $\hat{v} < \overline{v}$, this entry rule is not the unique equilibrium in the entry stage.

4. CONCLUDING REMARKS

In the presence of commitment failures, the equilibrium for an auction in which the acquisition of the good by one agent induces negative externalities on his competitors, entails strategic non participation. Moreover, if the level of the externality is low, the seller can select an auction procedure such that almost all potential bidders decide to participate and get an expected revenue close to the one she would obtain if she were able to commit. By contrast, when the externality is high, there is a coordination problem in the entry stage so that it is difficult to predict the level of participation. In particular, all potential bidders can decide not to show up. Therefore, the expected revenue of the seller can be drastically decreased with respect to the full commitment case. This result suggests that the degree of commitment of the seller to a particular auction procedure is crucial. Both the incentives to participate of potential bidders and the level of the expected revenue are very sensitive to the seller's commitment abilities.

From a theoretical perspective, we have shown that the optimal mechanism approach is not always helpful to predict the behavior of agents in real life situations. We have also shown that strategic non-participation occurs if the set of mechanisms is restricted due to commitment failures. Commitment failures affect the equilibrium outcome depending on the size of the externalities. When externalities are small, the solution is 'similar' to the solution obtained under full commitment. When externalities are large, the solution differs qualitatively and quantitatively.

It is important to note that strategic non participation and the subsequent result of screening of agents (only those with low valuations do not show up) is not the result of the presence of negative externalities. It comes from the combination of sufficiently large negative externalities and commitment failures leading to a restriction of the set of mechanisms. In other words, restricting the set of possible mechanisms is a *necessary condition* to get strategic non participation. This can emerge naturally in models of negative externalities where a particular type of suboptimal auction is considered (see Jehiel and Moldovanu, 1996; or Jehiel and Moldovanu, 2000). However, this is not a sufficient condition. It is always possible to restrict to suboptimal procedures that generate full participation, e.g., by making all reserve prices equal to an arbitrary value (possibly $\underline{v}$) and/or not including entry fees (see Jehiel and Moldovanu, 2000). Our analysis suggests that it is important to determine the allocation mechanisms that correspond to the actual commitment abilities in order to provide accurate predictions and prescriptions in particular economic situations.

APPENDIX

Proof of Proposition 1

Consider a first-price sealed bid auction with n bidders whose valuations are independently drawn from a distribution $G(\cdot)$. Assume the reserve price is r. Agent i anticipates that agent $j \neq i$ bids $b(v_j)$ where $b(\cdot)$ is a monotonic increasing function.

If i announces b_i and gets the good, his surplus is $v_i - b_i$. If he does not get it, his surplus is $-\alpha$ if another agent gets the good. The probability of winning is equal to the probability that the $n-1$ other agents have valuations v_j such that $b(v_j) < b_i$, i.e., $G(b^{-1}(b_i))^{n-1}$. Moreover, the probability of getting a surplus equal to $-\alpha$ is the probability that at least one agent among $n-1$ has a valuation v_j such that $b(v_j) > b_i$ and $b(v_j) > r$, i.e., $1-G(b^{-1}(b_i))^{n-1}$ if $b_i > r$ and $1-G(b^{-1}(r))^{n-1}$ if $b_i < r$. Let $u_i(v_i, b_i)$ be the expected utility of agent i when his valuation is v_i and he bids b_i,

$$u_i(v_i,b_i)=\begin{cases}(v_i-b_i)G(b^{-1}(b_i))^{n-1}-\alpha(1-G(b^{-1}(b_i))^{n-1}) & b_i \geqslant r\\ -\alpha(1-G(b^{-1}(r))^{n-1}) & b_i < r\end{cases}$$

Provided i bids above r, he chooses b_i such that $\partial/\partial b_i \; u_i(v_i, b_i) = 0$. Differentiating $u_i(v_i, b_i)$ with respect to v_i and using the previous optimality condition, we get:

$$\frac{\mathrm{d}u_i}{\mathrm{d}v_i} = \frac{\partial u_i}{\partial v_i} = G(b^{-1}(b_i))^{n-1} \geqslant 0 \tag{1}$$

In particular, there exists v_r such that an agent with that valuation bids exactly r. At the Nash equilibrium (and given symmetry) we must have $b_i = b(v_i)$. We also have $r = b(v_r)$. Substituting this in (1), we have $\mathrm{d}u_i/\mathrm{d}v_i = G(v_i)^{n-1}$ for all $v_i \geqslant b^{-1}(r) = v_r$. Then:

$$u_i(v_i, b_i) = \int_{b^{-1}(r)}^{v_i} G(s)^{n-1}\mathrm{d}s + k$$

where $k = u_i(v_r, r) = (v_r - r)G(b^{-1}(r))^{n-1} - \alpha(1-G(b^{-1}(r))^{n-1})$. The optimal bidding strategy must also be such that $u_i(v_r, r) =$

$-\alpha(1 - G(b^{-1}(r))^{n-1})$. Indeed, if $v_r > r$, any agent with valuation $v_i \in (r, v_r)$ could increase his utility by bidding r; if $v_r < r$, an agent with valuation v_r is worse-off by bidding r rather than bidding below r. Hence $v_r = b(r) = r$. Overall, we have both

$$u_i(v_i, b_i) = \begin{cases} (v_i - b_i)G(v_i)^{n-1} - \alpha(1 - G(v_i)^{n-1}) & v_i \geqslant r \\ -\alpha(1 - G(r)^{n-1}) & v_i < r \end{cases}$$

$$u_i(v_i, b_i) = \begin{cases} \int_r^{v_i} G(s)^{n-1}\mathrm{d}s - \alpha(1 - G(r)^{N-1}) & v_i \geqslant r \\ -\alpha(1 - G(r)^{n-1}) & v_i < r \end{cases}$$

Then, for all $v_i \geqslant r$, the optimal bid is given by:

$$\begin{aligned} &(v_i - b_i)G(v_i)^{n-1} - \alpha(1 - G(v_i)^{n-1}) \\ = &\int_r^{v_i} G(s)^{n-1}\mathrm{d}s - \alpha(1 - G(r)^{n-1}). \end{aligned}$$

When $v_i < r$, the agent makes an irrelevant bid below r (or does not bid at all). We get the result by replacing $G(\cdot)$ by $F(\cdot|\hat{v})$ and r by $\max(r_n, \hat{v})$. □

Proof of Proposition 2

The ex ante expected revenue is

$$E[R(\hat{v}, (r_n)_{n=1,\dots,N}, c)] = \Sigma_{n=1}^{N} p_n(\hat{v})(R_n(\hat{v}, r_n) + nc) \qquad (2)$$

LEMMA 3. *Reserve prices are such that $r_n \geqslant \hat{v}$ for all $n > 1$ and $r_1 \leqslant \hat{v}$.*

Proof. In an auction with $n \geqslant 2$ actual bidders and $r_n < \hat{v}$, the strategies played by bidders do not depend on r_n: the relevant reserve price is $\hat{v}$. Overall, the equilibrium procedure chosen by the seller is such that $r_n \geqslant \hat{v}$, $\forall n > 1$. Last, if $r_1 > \hat{v}$, all agents with valuations smaller than $\hat{v}$ have the same utility than an agent with valuation $\hat{v}$. Then $\hat{v}$ is not an equilibrium. □

For any mechanism $\mathcal{A}$, the entry fee is simply $c = H(\hat{v}, \hat{v}; \mathcal{A})$ (Definition 1):

$$\begin{aligned} c \;=\; &\pi_0(\hat{v})(\hat{v} - r_1) + \Sigma_{n=1}^{N-1}\alpha\pi_n(\hat{v})F(r_{n+1}|\hat{v})^n \\ &-\Sigma_{n=2}^{N-1}\alpha\pi_n(\hat{v})F(r_n|\hat{v})^n \end{aligned} \qquad (3)$$

Replacing in (2), the expected revenue is simply

$$\begin{aligned} E[R(\hat{v}, (r_n)_{n=2,\dots,N})] = &\textstyle\sum_{n=2}^{N} \tau_n(\hat{v}) \int_{r_n}^{\overline{v}} [F(v_i) \\ &-F(\hat{v})]^{n-1} \left[v_i - \alpha(n-1) - \frac{1-F(v_i)}{f(v_i)} \right] f(v_i) dv_i \\ &+ \textstyle\sum_{n=3}^{N} \tau_n(\hat{v}) \alpha (1-F(\hat{v})) \left[[1-F(\hat{v})]^{n-1} \right. \\ &\left. -[F(r_{n-1}) - F(\hat{v})]^{n-1} \right] \\ &+ \tau_1(\hat{v})(1-F(\hat{v}))\hat{v} + \alpha \tau_1(\hat{v})(1-F(\hat{v})) \end{aligned} \tag{4}$$

where $\tau_n(\hat{v}) = n \ p_n(\hat{v})/(1-F(\hat{v}))^n$. The seller maximizes $E[R(\hat{v}, r_2, \dots, r_N)]$ with respect to $r_2, \dots, r_N$. The first-order condition for r_n with $2 \leq n < N$ is:

$$\begin{aligned} &-\tau_n(\hat{v})(F(r_n) - F(\hat{v}))^{n-1} f(r_n) \left[r_n - \alpha(n-1) - \frac{1-F(r_n)}{f(r_n)} \right] \\ &-\alpha n \tau_{n+1}(1 - f(\hat{v}))(F(r_n) - F(\hat{v}))^{n-1} f(r_n) = 0 \end{aligned}$$

Since $\tau_{n+1}(\hat{v}) = \tau_n(\hat{v}) \ (N-n)/nF(\hat{v})$, the condition is equivalent to:

$$\begin{aligned} &\tau_n(\hat{v})(F(r_n) - F(\hat{v}))^{n-1} f(r_n) [r_n - \alpha(n-1) \\ &- \frac{1-F(r_n)}{f(r_n)} + \alpha(N-n) \frac{1-F(\hat{v})}{F(\hat{v})}] = 0 \end{aligned} \tag{5}$$

The first-order condition is satisfied in $\hat{v}$ and in $\tilde{r}_n(\hat{v})$ such that:

$$\tilde{r}_n(\hat{v}) - \alpha(n-1) - \frac{1-F(\tilde{r}_n(\hat{v}))}{f(\tilde{r}_n(\hat{v}))} = -\alpha(N-n) \frac{1-F(\hat{v})}{F(\hat{v})} \tag{6}$$

The second-order condition is:

$$\begin{aligned} &-\tau_n(\hat{v})(F(r_n) - F(\hat{v}))^{n-1} \frac{\mathrm{d}}{\mathrm{d}r_n} \\ &\left[r_n - \alpha(n-1) - \tfrac{1-F(r_n)}{f(r_n)} \right] f(r_n) < 0 \end{aligned}$$

then we have a maximum.

When $n = N$, the first-order condition is

$$-\tau_n(\hat{v})(F(r_N) - F(\hat{v}))^{N-1} f(r_N)[r_N - \alpha(N-1)$$
$$-\frac{1 - F(r_N)}{f(r_N)}] = 0$$

The solution is $\hat{r}_N(\hat{v}) = \max\{\hat{v}, \tilde{r}_N(\hat{v})\}$ where $\tilde{r}_N(\hat{v}) = r^*(\alpha, N)$.

LEMMA 4. *The optimal reserve price when* $n \in \{2, ..., N\}$ *is* $\hat{r}_n(\hat{v}) = \max\{\hat{v}, \tilde{r}_n(\hat{v})\}$ *with* $\tilde{r}_n(\hat{v})$ *increasing in n. Besides, it increases in* α.

Proof. If $\tilde{r}_n(\hat{v}) < \hat{v}$, the reserve price is $\hat{v}$ and for all $\hat{v}$, $\tilde{r}_n(\hat{v}) > \tilde{r}_{n-1}(\hat{v})$. Consider

$$g_n(v) = v - \alpha(n-1) - \frac{1 - F(v)}{f(v)}$$

$g_n(\cdot)$ is increasing in v and decreasing in n. Moreover, if $\hat{v}$ is such that $g_n(\underline{v}) > -\alpha(N-n)\ (1 - F(\hat{v}))/F(\hat{v})$, there is no solution satisfying (6). For all n, there exists a unique a_n such that

$$\underline{v} - \alpha(n-1) - \frac{1}{f(\underline{v})} = -\alpha(N-n)\frac{1 - F(a_n)}{F(a_n)}$$

and for all $\hat{v} > a_n$, there exists $\tilde{r}_n(\hat{v})$ satisfying (6). Note that a_{n+1} satisfies:

$$\underline{v} - \alpha n - \frac{1}{f(\underline{v})} = -\alpha(N-n-1)\frac{1 - F(a_{n+1})}{F(a_{n+1})}$$

Since $g_n(\underline{v}) > g_{n+1}(\underline{v})$, then:

$$-\alpha(N-n)\frac{1 - F(a_n)}{F(a_n)} < -\alpha(N-n-1)\frac{1 - F(a_{n+1})}{F(a_{n+1})}$$
$$-\alpha(N-n)\frac{1 - F(a_{n+1})}{F(a_{n+1})}$$

Consider also $h_n(\hat{v}) = \alpha(N-n)\ (1 - F(\hat{v}))/F(\hat{v})$. It is decreasing in $\hat{v}$ and in n . Combining the previous points, we have $a_n > a_{n+1}$. Last given $h(\hat{v})$ is decreasing in $\hat{v}$, $\tilde{r}_n(\hat{v})$ is increasing in $\hat{v}$ if it exists. Besides, $lim_{\hat{v} \to \overline{v}}\ \tilde{r}_n(\hat{v}) = r(\alpha, n)$ where $r(\alpha, n)$ satisfies:

$$r(\alpha, n) - \alpha(n-1) - \frac{1 - F(r(\alpha, n))}{f(r(\alpha, n))} = 0.$$

Overall, for all $\hat{v}$ and for all n, the optimal reserve price is $\hat{r}_n(\hat{v}) = \max(\hat{v}, \tilde{r}_n(\hat{v}))$. Given the properties of $g_n(\cdot)$ and $h_n(\cdot)$, if there exists n_1 such that $\hat{r}_{n_1}(\hat{v}) = \hat{v}$, then for all $n < n_1$, $\hat{r}_n(\hat{v}) = \hat{v}$. Similarly, if there exists n_2 such that $\hat{r}_{n_2}(\hat{v}) = \tilde{r}_{n_2}(\hat{v})$, then for all $n > n_2$, $\tilde{r}_n(\hat{v}) > \tilde{r}_{n_2}(\hat{v})$ and $\hat{r}_n(\hat{v}) = \tilde{r}_n(\hat{v})$.

Last, differentiating (6) with respect to α, we get:

$$\frac{\mathrm{d}}{\mathrm{d}v}\left(v - \frac{1-F(v)}{f(v)}\right)\Bigg|_{v=\tilde{r}_n} \frac{\mathrm{d}\tilde{r}_n}{\mathrm{d}\alpha} = n - 1 - (N-n)\frac{1-F(\hat{v})}{F(\hat{v})}$$

Since $n-1-(N-n)\ (1-F(\hat{v}))/F(\hat{v})$ is increasing in $\hat{v}$, positive in $\overline{v}$ and negative in $\underline{v}$, for all n, there exists b_n such that for all $\hat{v} > b_n$ (resp. $\hat{v} < b_n$), $\tilde{r}_n$ is increasing (resp. decreasing) in α. Moreover a_n is such that $n-1-(N-n)\ (1-F(a_n))/F(a_n) > 0$. Hence $a_n > b_n$. Thus $\tilde{r}_n$ exists if and only if $\hat{v} > a_n > b_n$ and it is increasing in α. □

LEMMA 5. *When* $\hat{v} = \underline{v}$, *the optimal procedure is* $\mathcal{A}^{**}$ *where* $\hat{r}_N(\underline{v}) = r^*(\alpha, N)$, $\hat{r}_n(\hat{v}) = \underline{v}$ *for all* $n = \{2, .., N-1\}$ *and* $\hat{r}_1 = \underline{v}$. *The entry fee is* c^*.

Proof. When $\hat{v} \to \underline{v}$, the solution of (5) is $\hat{r}_n(\underline{v}) = \underline{v}$ for all $n \in \{2, ..., N-1\}$. Given Lemma 3, $\hat{r}_1 = \underline{v}$. The first-order condition with respect to r_N yields $\hat{r}_N(\underline{v}) = r^*(\alpha, N)$. Last, replacing the reserve prices in c, the entry fee is $\hat{c} = \alpha F(\hat{r}_N(\underline{v})|\underline{v})^{N-1} \equiv c^*$. □

Combining the previous results, we get Proposition 2. □

Proof of Proposition 3

Consider an entry rule $\hat{v} < r^*(\alpha, N)$. An agent with valuation $\hat{v}$ who decides not to show up suffers the externality with probability $1 - \pi_0(\hat{v}) = 1 - F(\hat{v})^{N-1}$, then

$$U_i^{ne}(\hat{v}, \hat{v}; \mathcal{A}^{**}) = -\alpha(1 - F(\hat{v})^{N-1})$$

If he participates, he wins if and only if no other agent participates; he suffers the externality for sure if the number of his competitors is between 1 and $N-2$; he suffers the externality if one of them has a valuation greater than the reserve price when all enter. Then

$$\begin{aligned} U_i^{e}(\hat{v}, \hat{v}; \mathcal{A}^{**}) \;=\; & F(\hat{v})^{N-1}(\hat{v} - \underline{v}) - \alpha \textstyle\sum_{n=1}^{N-2} \pi_n(\hat{v}) \\ & -\alpha\pi_{N-1}(\hat{v})[1 - F(r^*(\alpha, N)|\hat{v})^{N-1}] \end{aligned}$$

Consider now an entry rule $\hat{v} > r^*(\alpha, N)$, the expected utility of the agent is the same as in the previous case when he does not participate. If he participates, his expected utility becomes

$$U_i^e(\hat{v}, \hat{v}; \mathcal{A}^{**}) = F(\hat{v})^{N-1}(\hat{v} - \underline{v}) - \alpha \sum_{n=1}^{N-1} \pi_n(\hat{v})$$

Let $g(\hat{v}; \mathcal{A}^{**}) \equiv H(\hat{v}, \hat{v}; \mathcal{A}^{**}) - c^\star$, it is:

$$\begin{cases} -\alpha F(r^*(\alpha, N))^{N-1} + F(\hat{v})^{N-1}(\hat{v} - \underline{v}) \\ \quad +\alpha(F(r^*(\alpha, N)) - F(\hat{v}))^{N-1} & \hat{v} < r^*(\alpha, N) \\ -\alpha F(r^*(\alpha, N))^{N-1} + F(\hat{v})^{N-1}(\hat{v} - \underline{v}) & \hat{v} \geq r^*(\alpha, N) \end{cases}$$

This function is increasing in $\hat{v}$ on $[r^*(\alpha, N), \overline{v}]$ and it is 0 in $\underline{v}$. Given Definition 2, $\underline{v}$ is the unique REE if and only if $g(\hat{v}; \mathcal{A}^{**}) > 0, \forall\, \hat{v} > \underline{v}$.

LEMMA 6. *$\hat{v} = \underline{v}$ is not the unique REE.*

Proof. Note that

- $r^*(\alpha, N)$ is increasing in α and $r^*(\alpha, N) = \overline{v}$ if $\alpha > \alpha^\star$, where $\alpha^\star = \frac{\overline{v}}{N-1}$. Moreover, $r^*(0, N) > \underline{v}$.
- There exists $\alpha_N^\star$ such that $\forall \alpha < \alpha_N^\star$, $r^*(\alpha, N) > \underline{v} + \alpha$ and $\forall \alpha > \alpha_N^\star$, $r^*(\alpha, N) < \underline{v} + \alpha$.
- $g(r^*(\alpha, N); \mathcal{A}^{**}) > 0$ (resp. < 0) if $r^*(\alpha, N) > \underline{v} + \alpha$ (resp. $< \underline{v} + \alpha$) .
- For all $\hat{v} < r^*(\alpha, N)$, $g(\hat{v}; \mathcal{A}^{**}) < F(\hat{v})^{N-1}(\hat{v} - \underline{v} - \alpha)$. Then, for all $\hat{v} < \underline{v} + \alpha$, we have $g(\hat{v}; \mathcal{A}^{**}) < 0$.

Combining the previous points:

(i) $\forall\, \alpha > \alpha_N^\star$, $\overline{v}$ is a REE if $g(\overline{v}; \mathcal{A}^{**}) < 0$. Otherwise, there exists $\hat{\hat{v}} \in (\underline{v} + \alpha, \overline{v})$ such that $g(\hat{\hat{v}}; \mathcal{A}^{**}) = 0$ and $\hat{\hat{v}}$ is a REE. Overall $\underline{v}$ and $\hat{\hat{v}}$ or $\overline{v}$ are REE.

(ii) $\forall\, \alpha < \alpha_N^\star$, and for all $\hat{v} < \underline{v} + \alpha$, $g(\hat{v}; \mathcal{A}^{**}) < 0$. Given that $g(r^*(\alpha, N); \mathcal{A}^{**}) > 0$, there exists $\hat{\hat{v}} \in [\underline{v} + \alpha, r^*(\alpha, N)]$ such that $g(\hat{\hat{v}}; \mathcal{A}^{**}) = 0$. Overall, $\underline{v}$ and $\hat{\hat{v}}$ are REE. □

LEMMA 7. *The entry rule $\underline{v}$ is never Pareto dominant.*

Denote by $\hat{v}_2$ an REE different from $\underline{v}$ and consider an agent with valuation $v_i < \min\{\hat{v}_2, r^*(\alpha, N)\}$. If bidders coordinate on $\underline{v}$, then his expected utility is

$$U_i^e(v_i, \underline{v}; \mathcal{A}^{**}) = -\alpha.$$

If by contrast they coordinate on $\hat{v}_2$, his expected utility is

$$U_i^{ne}(v_i, \hat{v}_2; \mathcal{A}^{**}) > -\alpha.$$

Then $\underline{v}$ is not Pareto dominant. □

Combining the two previous lemmas, we get Proposition 3. □

Proof of Proposition 4

Suppose the seller offers a mechanism $\hat{A}^*(\hat{v}^\star, r_1)$ corresponding to the entry rule $\hat{v}^*$ with reserve prices $\hat{r}_k(\hat{v}^*)$ for all $k \in [2, ..., N]$ satisfying point (i) in Proposition 2, $r_1 \in [\underline{v}, \hat{v}^*]$ and $\hat{c}(r_1)$ satisfying point (iii) in Proposition 2. Given this mechanism, let $g(\hat{v}, r_1) \equiv g(\hat{v}; \hat{A}^*(\hat{v}^\star, r_1)) \equiv H(\hat{v}, \hat{v}; \hat{A}^*(\hat{v}^\star, r_1)) - \hat{c}(r_1)$. Given Definition 2, $\hat{v}^\star$ is the unique REE if r_1 is such that $g(\hat{v}, r_1) = 0$ if and only if $\hat{v} = \hat{v}^*$. The function $g(\hat{v}, r_1)$ is continuous but not differentiable with respect to $\hat{v}$. Formally,

– If $\hat{v} < r_1$, $g_1(\hat{v}, r_1) = -\hat{c}(r_1) + \sum_{n=1}^{N-1} \alpha\pi_n(\hat{v})[F(\hat{r}_{n+1}(\hat{v}^*)|\hat{v})^n - F(\hat{r}_n(\hat{v}^*)|\hat{v})^n]$

– If $\hat{v} \in [\hat{r}_{k-1}(\hat{v}^*); \hat{r}_k(\hat{v}^*)]$ with $2 \leq k < N$: $g_k(\hat{v}, r_1) = -\hat{c}(r_1) + \pi_0(\hat{v})(\hat{v} - r_1) + \sum_{n=k-1}^{N-1} \alpha\pi_n(\hat{v})F(\hat{r}_{n+1}(\hat{v}^*)|\hat{v})^n - \sum_{n=k}^{N-1} \alpha\pi_n(\hat{v})F(\hat{r}_n(\hat{v}^*)|\hat{v})^n$

– If $\hat{v} \in [\hat{r}_N(\hat{v}^*); \overline{v}]$: $g_{N+1}(\hat{v}, r_1) = -\hat{c}(r_1) + \pi_0(\hat{v})(\hat{v} - r_1)$

LEMMA 8. *When α is sufficiently small, an equilibrium with minimal screening can be obtained.*

Note that

- $\partial g_k/\partial r_1 = \pi_0(\hat{v}^*) - \pi_0(\hat{v})$ for all k. It is positive for all $\hat{v} < \hat{v}^*$ and negative for all $\hat{v} > \hat{v}^*$.
- When $\alpha \to 0$ and for all $r_1 < \hat{v}^*$ we have $g_1(\hat{v}, r_1) = -\hat{c}(r_1)$ and for all $k > 2$, $g_k(\hat{v}, r_1) = -\hat{c}(r_1) + \pi_0(\hat{v})(\hat{v} - r_1)$. Moreover, $\partial g/\partial \hat{v} > 0$ for all $\hat{v} > \underline{v}$. Last, $g(\underline{v}, r_1) < 0$ and $g(\overline{v}, r_1) >$

0. Then, when α tends to 0, any target $\hat{v}^{\star} > \underline{v}$ is the unique REE for all $r_1 < \hat{v}^*$.

Combining the two previous points and using a continuity argument, for all α sufficiently small there exist targets $\hat{v}^{\star}$ close to $\underline{v}$ and corresponding procedures $\hat{\mathcal{A}}^*(\hat{v}^*, r_1)$ such hat $\hat{v}^*$ is the unique REE. This is the case provided the seller decreases r_1 so that $g(\hat{v}, r_1) < 0$ for all $\hat{v} < \hat{v}^*$ and $g(\hat{v}, r_1) > 0$ for all $\hat{v} > \hat{v}^*$. □

The second part of Proposition 4 is obtained by noting that if α becomes sufficiently large, for all $\hat{v}^*$ and $\hat{A}^*(\hat{v}^{\star}, r_1)$, $g_{N+1}(\overline{v}, r_1) < 0$, then $\overline{v}$ is always an equilibrium. □

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NOTES

1. Klibanoff and Morduch (1995) and Carrillo (1998) study optimal regulation with costly transfers and externalities that are type-independent and type-dependent respectively.
2. See Engelbrecht-Wiggans (1980) and McAfee and McMillan (1987a) for surveys. See also Myerson (1981) for his seminal paper on optimal auction.
3. See also Brocas (2002a) for an analysis of the optimal licensing of an innovation when the acquisition of the item by one producer induces externalities on his competitors.
4. Jehiel et al. (1996) focus on common knowledge identity-dependent externalities. See also Jehiel et al. (1999) for the case where the private information of an agent is the vector of his payoffs should any of other buyers get the good (the analysis becomes a multidimensional mechanism design problem). See also Brocas (2002b) where externalities are type-dependent instead of identity-dependent.
5. Identity-dependency does not introduce qualitative departures from the symmetric case. For simplicity, we restrict to symmetric and observable externalities in this analysis.

6. See Klemperer (2000) for an analysis of the relevance of auction theory to study a large variety of economic situations. He also stresses the fact that many markets are nowadays literally auction markets, such as spectrum rights markets and treasury bill auctions.
7. Note that our investigation departs from other analyses of two-step procedures where screening is avoided by assuming that agents do not know their valuations before entering the auction. See Levin and Smith (1994), McAfee and McMillan (1987b) for analyses of entry with ex ante symmetric beliefs on future valuations.
8. The case of non-common knowledge externalities have been investigated in the literature. See Brocas (2002b) for an analysis of the optimal auction when externalities depend on (privately known) valuations. We abstract from the effects generated in that paper to focus on the issue of commitment.
9. Jehiel et al. (1999) obtain a similar result in their multidimensional framework.
10. The problem at hand differs from previous works where the outside option is fixed and exogenous or where is is an exogenous function of the type (as in standard models of countervailing incentives). In this model, the outside option is fixed but mechanism-dependent.
11. We would get the same results with a second-price sealed bid auction.
12. Our analysis has the flavor of Samuelson (1985) where agents need to sink preparation costs to participate in the auction: entry fees, as preparation costs, distort the entry strategy and an agent with a low valuation and a low probability of winning has incentives not to show up.
13. When $\alpha = 0$, the bidding strategy in the first price sealed bid auction is

$$\beta(v_i) = v_i - \frac{\int_y^{v_i} F(s|\hat{v})^{n-1} \mathrm{d}s}{F(v_i|\hat{v})^{n-1}}.$$

It is interesting to note that $v_i + \alpha$ cannot be interpreted as a modified valuation for the good. If it were the case, it would be sufficient to replace v_i by $v_i + \alpha$ in $\beta(\cdot)$. The reason why the expression we would then obtain does not correspond to the expression in Proposition 1 is simply that only v_i is private information.
14. Note also that $b(v_i)$ is increasing in v_i, so it is an equilibrium strategy.
15. In a second-price sealed bid auction, the optimal bid is $b_i(v_i) = v_i + \alpha$, as in Lemma 2.

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